Designing a side-emitting lens using the composing method

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Abstract

The paper considers a design method for a side-emitting lens working with a single LED source and providing a narrow light beam in the horizontal direction within 360°. The authors deal with the composing method in the design, which is usually used to synthesize the initial scheme of the imaging systems. However, in this case, a similar approach is applied to the synthesis of a system, the task of which is to provide a certain shape and characteristics of the light beam. The stage of choosing an initial principal design and synthesis for non-imaging optics is especially important since this area is characterized by the use of so-called local optimization, the result of which strongly depends on the original system. Therefore, the stage of forming an initial approximation of the system becomes crucial. In this case, the composing method can provide the most effective choice of an initial scheme of a side-emitting lens. The application of the composing method and the theory of aberrations in the field of synthesis of the initial design of a side-emitting lens is shown. The paper describes a method for selecting the key parameters of the system, and presents the relationships that allow a preliminary evaluation of the characteristics of the designed system without the use of time-consuming calculation or optimization procedures. The presented approach makes it possible to ensure the choice of the initial point of the system for further optimization, as well as to achieve high efficiency of using the light flux by the optical system of the lens (up to 90%), only due to the optimal size of the zones into which the beam is divided and to the optimal parameters of the generating curves. In this case, the lens profile is formed by two zones in each the profile is a conic curve, hence, the curve can be described by a small number of parameters, which is very convenient at the stage of searching the initial scheme. The proposed approach can be applied in the design of such systems, as well as applied at the stage of preliminary assessment of achievable characteristics, which can significantly speed up the development process.

Keywords
non-imaging optics, composing method, aberration theory, side-emitting lens, LED light sources

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Разработка линзы бокового свечения с использованием метода композиции

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Аннотация

Предмет исследования. Рассмотрен метод проектирования линзы бокового свечения, работающей с единичным светодиодным источником света и обеспечивающей формирование узкого светового пучка в горизонтальном направлении в пределах 360°. Метод. При проектировании использован метод композиции, который применяется...
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**Introduction**

LED sources are used everywhere to illuminate architectural objects, posters, shop windows, advertisements, for road illumination, in lighthouses, signal lighting, and other lighting devices [1].

The expansion of the fields of LEDs application has led to the rapid development of design methods for such systems. Most methods are based on the direct solution of the problem of finding the surface profile with the known characteristics of the light source or on automated design methods using specialized software (ASAP, LightTools, Zemax).

In practice, optical engineers widely use the selection of the shape of surfaces and elements based on approximate requirements for the ray paths and methods of automated design (optimization) of optical systems. However, even when applying this approach, it is necessary to have an idea of what shape the lens should be (one or more surfaces, the presence of total internal reflection, the division of the beam into zones, etc.). Besides, when using the SMS (Simultaneous Multiple Surface) method [2–4] or ray-mapping method [5], there is no way to preliminarily evaluate the achievable characteristics (for example, beam divergence when designing a collimator). Therefore, even for a standard element in the form of a collimating system operating on the Total Internal Reflection (TIR) effect, there are many solutions and methods proposed by many scientists [6–8].

For lenses and systems operating with one or multiple sources and providing a narrow beam in an annular zone for the full azimuth angle of 360°, methods based on finding the surface profile using the ray-mapping technique have also been proposed in a number of works [9, 10].

An alternative method of designing optics, which allows finding an initial solution quickly enough, is the so-called composing method. The composing method is understood as combining elements (optical surfaces or lenses) with studied and/or well-known properties. For imaging optics, these properties are aberration properties, but it is necessary to take into account the achievable distribution of luminous intensity or illumination distribution for non-imaging systems [11, 12].

As a rule, aberration theory is not applicable for the case of a source emitting inside a wide angle. To apply it to the area of non-imaging systems, it is necessary to consider not the aberration properties, but their influence on the light beam characteristics. For a collimating lens or a side-emitting lens these properties are the possible residual divergence.

Collimating optics can act as the basic version of the system making it possible to confirm the effectiveness of the proposed approach [12]. The authors have previously considered the application of the composing method for the initial scheme synthesis and analysis of the properties of a collimating lens operating on the effect of TIR. In this work, it is proposed to apply this approach for designing a side-emitting lens that provides a horizontal narrow light beam within azimuth angle 360°.

**The composing method for designing a side-emitting lens**

Let us consider at the first stage a point light source emitting within a hemisphere and a schematic layout of a side-emitting lens. Below, when composing a side-emitting lens, we consider several basic solutions, the properties of which are well known from the theory of geometric aberrations [13–15]:
— a lens with a first convex hyperboloid surface facing a point source and a second flat surface. The conic constant of the profile (hyperbola) \( k = -n^2 \). In the case of a point source located in the focal plane of such a lens, it provides an ideal parallel beam;
— a lens with a first spherical surface, concentric to the source point, and a second ellipsoid surface. The conic constant for the profile (ellipse) \( k = -1/n^2 \);
— parabolic mirror surface. Provides a parallel beam when a point source is located at the focal point;
— a spherical surface. It does not change the divergence of the beam if the center of the beam is in its center of curvature;
— a flat surface. It does not change the divergence of a parallel beam of rays incident perpendicular to it.

It should be noted that the elements listed above provide a strict absence of spherical aberration, that is, a strict stigmatic beam.

Since it is necessary to provide a horizontal parallel beam for the case when the LED light source radiates into the upper hemisphere, several solutions are possible using the elements listed above. Relatively powerful LED light sources often emit within a hemisphere, and according to the elements listed above, relatively powerful LED light sources are also used in zone I, but zone II is a plane-hyperbolic lens, into which an additional flat surface is introduced, providing the corresponding direction of the beam axis. Rays that correspond zone II are depicted in a red color, the edge ray for zone I is shown in blue.

A plane-hyperboloid lens for zone I is not considered since it will not be possible to manufacture the configuration obtained in this way by molding, and the variant with a sphero-elliptical lens for zone II is not analyzed, since in this case the dimensions of the system significantly increase or it is impossible to ensure full capture of the light beam in zone II.

For a lens of the first type (Fig. 1, a), the residual beam divergence in the horizontal direction will be determined by the source size, as well as by the coma aberration. For the central zone, the divergence angle due to the source size (\( \omega'_I \)) and aberrational (due to coma) (\( \omega'_I \text{com} \)) can be determined on the basis of geometric relations and formulas of the theory of the third-order aberrations [16]:

\[
\omega'_I \approx \frac{y}{f'_I},
\]

\[
\omega'_I \text{com} = \frac{3y}{2(n-1)f'_I} \sin^2 \sigma_f,
\]

where \( y \) — size of the luminous area of the source; \( f'_I \) — focal length of a sphero-ellipsoidal lens; \( n \) — refraction index of the material.

Full beam divergence is defined by the full divergence angle that is the sum of the values:

\[
\omega'_I + \omega'_I \text{com}.
\]

Besides, it is convenient to introduce dimensional relationships for zone I. The diameter of the side-emitting

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**Fig. 1. Possible profiles for generating a side-emitting lens: a sphero-ellipsoidal lens and paraboloid surface (a); a sphero-ellipsoidal lens and plano-hyperboloidal lens with an additional flat surface (b)**
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Lens (lower part — zone I) is determined by the parameters of the spheroid-ellipsoidal lens. It can be shown that with this configuration, the focal length of a spheroid-ellipsoidal lens is described by the expression:

\[ f' = r_{sph} + d, \]

where \( r_{sph} \) — the radius of the spherical surface; \( d \) — lens thickness.

Vertex radius of the second surface (ellipsoidal):

\[ r_2 = \frac{f'(n-1)}{n} = \frac{(d + r_{sph})(n-1)}{n}. \]

If the radius of the spherical surface is set as the initial parameter (the minimum value is determined by the required dimensions), as well as the focal length of the lens \( f' \) selected based on the required beam divergence, then the lens thickness is determined almost unambiguously. On the other hand, on the contrary, it is possible to set the thickness of the lens and the radius of the spherical surface and analyze whether the desired lens characteristics are achievable in this case.

These expressions are valid for both types of lenses since zone I in all cases has the same shape. It should be noted that in this case, the energy distribution of the source is not taken into account, as well as the fact that the source area is tilted relative to the profile axis.

For zone II in a system of the first type, the divergence due to the size of the source \( \omega_{II} \) and the aberration divergence \( \omega_{II coma} \) are defined similarly:

\[ \omega_{II} = \frac{y}{f'_{II}}, \]
\[ \omega_{II coma} = \frac{3y}{4f'_{II}} \sin^2(\pi/2) - \sin^2\sigma_1 = \frac{3y}{4f'_{II}} \cos^2\sigma_1, \]

where \( f'_{II} \) — focal length of the paraboloid.

Lens dimensions (Fig. 1, a) could be described by the formulas:

\[ Y_{max} = \frac{2f'_{II}}{\tan \sigma_2} \left[ 1 + \sqrt{1 + \tan \sigma_1} \right], \]
\[ Z_{II} = \frac{y^2}{4f'_{II}} - f'_{II}. \]

Full divergence angle for the lens of the first type is the maximum of the two values — divergence for zone I and zone II.

For a lens of the second type (Fig. 1, b) for zone II, the relations for a plano-hyperboloidal lens are valid:

\[ \omega_{II} = \frac{3y}{2(n-1)f'_{IIp}}, \]
\[ \omega_{II coma} = \frac{3yn}{2(n-1)f'_{IIp}} \sin^2\sigma_1, \]

where \( f'_{IIp} \) — focal length of the plano-hyperboloidal lens.

As for the previous case, the full divergence angle for the lens of the second type is the maximum of the two values — divergence for zone I and zone II.

As can be seen from the analysis of the formulas, the lens of the first type provides a lower divergence, since the focal length of the paraboloid can be greater than the focal length of a plane hyperboloidal lens, which can be physically realizable in the structure of the side-emitting lens. Moreover, coma of a paraboloid is smaller (with the same characteristics). However, the second type could be more compact.

Design example of the side-emitting lens

Fig. 2 and 3 show the examples of lenses of the first and second types, built using the proposed approach, as well as the results of simulations performed in Zemax Optic Studio.

For the first type lens if we choose the focal length of paraboloidal part \( f'_{IIp} = 2.2 \) mm, \( Y_{max} = 13 \) mm, the focal length of a spheroid-ellipsoidal lens \( f' = 11.35 \) mm, the conic constant of ellipsoidal surface (for zone I) \( k = -0.444 \), the radius of a spherical surface \( r_{sph} = 4.35 \) mm we obtain the divergence angles \( \omega_{II} = 2.5°, \omega_{II coma} = 3.8° \), the full divergence angle for zone I \( \omega = 6.4° \), for zone II \( \omega_{II} = 13°, \omega_{II coma} = 4.9° \), the full divergence angle for zone II \( \omega = 8.1° \). It is worth noting that \( y = 0.5 \) mm was used in calculations, and the refraction index \( n = 1.5 \). The larger value of the divergence at the same sign determines the total divergence value. Thus we can see that residual divergence is more because of zone II. The second variant in Table presents the parameters of the lens of the second type — with the plano-hyperboloidal lens in zone II. The parameters of the profile for zone I are the same, and the parameters of the hyperboloid profile (for zone II) are as following: the focal length of the plano-hyperboloidal lens \( f'_{IIp} = 1.5 \) mm, the conic constant for the hyperboloid \( k_1 = -2.2253 \). The values of the divergence angles for the two variants are given in Table below.

Fig. 2, a presents the lens of the first type with the parameters from the first line in Table (variant 1). The distribution of the luminous intensity on the polar detector [17] in the system when working with a Lambertian light source having the dimensions of a luminous area 1 × 1 mm² is depicted in Fig. 2, b. The source flux in calculation equals 100 lm. The lens material is polymethylmethacrylate (PMMA). As can be seen from the figure, the lens provides the formation of an annular beam of rays, and about 91 % of the light flux is concentrated within an angle of ±8°. Table also provides the value of the Full Width on the Half Maximum of the beam (FWHM).

In this case, reflection losses were not taken into account. The results of simulations have shown good agreement with theoretical values. If in theoretical evaluations we use the diagonal size of the area source \( (y_d = 0.7) \) the theoretical values will be even closer to the results. Hence, here we demonstrated how the parameters of the profile curves influence the resulting properties of the beam.

Fig. 3, a demonstrates the 3D model of the lens for the second variant from Table. Fig. 3, b shows the distribution of the luminous intensity in the system working with a lambertian light source with the same parameters as in the previous example. As shown in Fig. 3, b, the lens provides the annular beam of rays; about 68 % of the light flux is concentrated within an angle of ±8° from the horizon (for an angle ±5°, the corresponding value is 60 % of the energy). In this case, when the divergence angles are
Table. Parameters for the curves of the lenses and their characteristics

<table>
<thead>
<tr>
<th>Variant</th>
<th>Parameters</th>
<th>Zone I</th>
<th>Zone II</th>
<th>Full divergence angle (theoretical)</th>
<th>Divergence angles in simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$f'<em>1 = 11.35$ mm,$r</em>{sph} = 4.35$ mm, $r_2 = 3.783$ mm, $k = -0.444$, $f'<em>{lph} = 2.2$ mm, $Y</em>{max} = 13$ mm</td>
<td>$\omega'<em>I = 2.5^\circ$, $\omega'</em>{Icoma} = 3.8^\circ$</td>
<td>$\omega'<em>II = 13^\circ$, $\omega'</em>{IIcoma} = -4.9^\circ$</td>
<td>$\omega' = 8.1^\circ$</td>
<td>$\pm 5^\circ$ (corresponding solid angle contains 83.5% of energy) $\pm 8^\circ$ (91% of energy) $\pm 15^\circ$ (95% of energy)</td>
</tr>
<tr>
<td>2</td>
<td>$f'<em>1 = 11.35$ mm,$r</em>{sph} = 4.35$ mm, $r_2 = 3.783$ mm, $k = -0.444$, $f'_{lph} = 1.5$ mm, $k_1 = -2.2253$</td>
<td>$\omega'<em>I = 2.5^\circ$, $\omega'</em>{Icoma} = 3.8^\circ$</td>
<td>$\omega'<em>II = 19^\circ$, $\omega'</em>{IIcoma} = 42^\circ$</td>
<td>$\omega' = 61^\circ$</td>
<td>$\pm 5^\circ$ (corresponding solid angle contains 60% of energy) $\pm 8^\circ$ (68% of energy) $\pm 15^\circ$ (76% of energy)</td>
</tr>
</tbody>
</table>

Fig. 2. The design example of a side-emitting lens of the first type: 3D lens model (a); light intensity distribution on a polar detector (b)

Fig. 3. The design example of a side-emitting lens of the second type: 3D lens model (a); light intensity distribution on a polar detector (b)
relatively large the correspondence between the theoretical values and simulation results is larger than the trigonometric functions are omitted in the formulas. However, the agreement is still good enough. As can be seen, the theoretical expressions provide a clear understanding of what zone and what parameters are responsible for the divergence, thus they provide ideas about the ways of improving the lens characteristics.

In accordance with theoretical calculations, the second type of lens has lower efficiency, but it has smaller dimensions: the maximum lens diameter is determined in this case by the dimensions of the lower zone (a spheroidal lens) and is equal to 22.7 mm.

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### References


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### Conclusion

In the work, it is proposed to compose the starting point of the side-emitting system for LED sources based on the ideas of the composing theory. Two possible configurations of the lens are considered, the relations that determine the collimation properties of a side-emitting lens (the size of the angular sector) are obtained. The work provides the formulas that allow generating a lens cross-sectional profile quickly without using complex mathematical relationships, taking into account overall constraints, as well as evaluating the properties of the system at the early design stage. An example is given to illustrate the proposed approach.