НАУЧНО-ТЕХНИЧЕСКИЙ ВЕСТНИК ИНФОРМАЦИОННЫХ ТЕХНОЛОГИЙ, МЕХАНИКИ И ОПТИКИ
март–апрель 2023 Том 23 № 2 http://ntv.ifmo.ru/
SCIENTIFIC AND TECHNICAL JOURNAL OF INFORMATION TECHNOLOGIES, MECHANICS AND OPTICS
March–April 2023 Vol. 23 No 2 http://ntv.ifmo.ru/en/
ISSN 2226-1494 (print) ISSN 2500-0373 (online)

doi: 10.17586/2226-1494-2023-23-2-263-270

Brief review of the development of theories of robustness, roughness and bifurcations of dynamic systems

Roman O. Omorov✉

Institute of Machine Science and Automation of the National Academy of Sciences of the Kyrgyz Republic, Bishkek, 720071, Kyrgyz Republic

romano_ip@list.ru✉, https://orcid.org/0000-0003-3555-1323

Abstract

The development issues of theories of robustness, roughness and bifurcations of dynamic systems are considered. In the modern theory of dynamic systems and automatic control systems, researches of the properties of roughness and robustness of systems are becoming more and more important. The work considers methods of research and ensuring robust stability of interval dynamic systems of both algebraic and frequency directions of robust stability. The main results of the original algebraic method of robust stability for continuous and discrete time are given. In the frequency direction of robust stability, the issues of a frequency-robust method to the analysis and synthesis of robust multidimensional control systems based on the use of the frequency condition number of the transfer matrix of the “input-output” ratio are considered. The main provisions of the theory and method of topological roughness of dynamic systems based on the concept of roughness according to Andronov-Pontryagin are presented with the introduction of a measure of roughness of systems in the form of a condition number of matrices of reduction to a diagonal (quasi-diagonal) basis at special points of phase space. Criteria for dynamic systems bifurcations are formulated. Applications of the topological roughness method to synergetic systems and chaos have been used to investigate many systems, such as Lorenz and Rössler attractors, Belousov-Jabotinsky, Chua systems, “predator-prey” and “predator-prey-food”, Hopf bifurcation, Schumpeter and Caldor economic systems, Henon mapping, and others. For research of weakly formalized and non-formalized systems, the use of the approach of analogies of theoretical-multiple topology and the abstract method to such systems is proposed. Further research suggests the development of roughness and bifurcation theories for complex nonlinear dynamical systems.

Keywords

method of topological roughness, condition number of a matrix, bifurcation of systems, robustness of control systems, interval dynamical systems, multidimensional control systems, frequency-robust method, frequency condition number, synergetic systems, chaos, special points and trajectories, Sylvester matrix equation

For citation: Omorov R.O. Brief review of the development of theories of robustness, roughness and bifurcations of dynamic systems. *Scientific and Technical Journal of Information Technologies, Mechanics and Optics*, 2023, vol. 23, no. 2, pp. 263–270. doi: 10.17586/2226-1494-2023-23-2-263-270

УДК 62-50

Краткий обзор развития теорий робастности, грубости и бифуркаций динамических систем

Роман Оморович Оморов✉

Институт машиноведения и автоматики Национальной академии наук Кыргызской Республики, Бишкек, 720071, Кыргызская Республика

romano_ip@list.ru✉, https://orcid.org/0000-0003-3555-1323

Аннотация

Рассмотрены вопросы развития теорий робастности, грубости и бифуркаций динамических систем. В современной теории динамических систем и систем автоматического управления все более важными становятся исследования свойств грубости и робастности систем. Изучены методы алгебраического и частотного направлений исследований и обеспечения робастной устойчивости интервальных динамических систем. Приведены основные результаты оригинального алгебраического метода робастной устойчивости для непрерывного и дискретного времени. В частотном направлении исследованы вопросы частотно-робастного метода анализа и синтеза робастных многомерных систем управления на основе использования частотного числа обусловленности передаточной матрицы отношения «вход–выход». Изложены основные положения теории и метода топологической грубости динамических систем. Положения основаны на понятии грубости по Андронову–Понтрягину с введением меры грубости систем в виде числа обусловленности матриц приведения к диагональному (квазидиагональному) базису в особых точках фазового пространства. Сформулированы критерии бифуркаций динамических систем. Приложения метода топологической грубости использованы для исследований синергетических систем и их хаоса на примерах: системы Лоренца и аттрактора Ресслера; реакции Белоусова–Жаботинского; системы Чуа; систем «хищник–жертва» и «хищник–жертва–пища»; бифуркации Хопфа; экономических систем Шумпетера и Калдора; отображения Энона и других. Для исследования слабо формализованных и неформализованных систем предложено использование подхода аналогий теоретико-множественных топологий и абстрактного метода к таким системам. Дальнейшее исследование предполагает развитие теорий грубости и бифуркаций для сложных нелинейных динамических систем.

Ключевые слова
метод топологической грубости, число обусловленности матрицы, бифуркация систем, робастность систем управления, интервальные динамические системы, многомерные системы управления, частотно-робастный метод, частотное число обусловленности, синергетические системы, хаос, особые точки и траектории, матричное уравнение Сильвестра

Ссылка для цитирования: Оморов Р.О. Краткий обзор развития теорий робастности, грубости и бифуркаций динамических систем // Научно-технический вестник информационных технологий, механики и оптики. 2023. Т. 23, № 2. С. 263–270 (на англ. яз.). doi: 10.17586/2226-1494-2023-23-2-263-270

Introduction

The interests of researchers who are attracted by the problems of robustness and roughness in various fields of science and technology [1–4], and not only in control theory, but also in ecology, synergetics, etc., are related to the fact that these problems relate to the most important properties of systems considered in their actual functioning. Especially it expands the boundaries of the problem of roughness, its connection with the problems of bifurcations and catastrophes.

As for control systems, the issues of robust stability are currently the most considered and solved. The solution of these issues is connected with the fundamental works of V.L. Kharitonov [5, 6] in which the issues of robust stability for interval polynomials are solved.

At present, many new results have been obtained in the theory of robust stability, first of all, the line theorem and discrete analogs, and variants of Kharitonov's theorems. Soviet and Russian scientists — Ya.Z. Tsypkin and B.T. Polyak [4], Yu.I. Neimark [7] developed frequency criteria for robust stability of the Mikhailov, Nyquist, and D-decomposition types.

The author's papers [8–10] present original results obtained for continuous and discrete linear interval dynamical systems which are generally called *the algebraic method of robust stability*.

The possibilities of matrix equations of the Sylvester type made it possible to take a fresh look at the traditional methods and tools for research multidimensional control systems, especially in the frequency direction of the theory of robustness for multidimensional systems. These include frequency transfer matrices and frequency characteristics constructed on their basis. In particular, the problem of designing frequency transfer matrices is solved using the concept of similarity of the forced component of the state of a multidimensional system to the state of the source of a finite-dimensional exogenous impact [11].

The concept of similarity made it possible, from a unified algorithmic standpoint, to construct frequency transfer matrices of continuous, discrete and multidimensional systems with modulation for single-frequency and multi-frequency cases of excitation of system inputs by harmonic exogenous action.

The use of the singular value decomposition of the frequency transfer matrices of multidimensional systems makes it possible to construct majorant and minorant amplitude and phase frequency characteristics of the studied systems by state, output, and error on the extremal elements of the algebraic spectrum of singular numbers and singular bases.

The main procedures of *the frequency-robust method* for the synthesis of multidimensional systems are based on the possibilities of the modal-robust generalized modal control.

In the classical formulation, the questions of *roughness and bifurcations* of dynamical systems were posed at the dawn of the formation of topology as a new scientific direction in mathematics by the great French mathematician and physicist H. Poincaré [1], in particular, the term bifurcation was first introduced by him and means literally "bifurcation" or, in other words, new solutions branch off from the solutions of the equations of dynamical systems.

Many fundamental results in the theory of roughness and bifurcations were obtained by A.A. Andronov and his school. In the work of A.A. Andronov, L.S. Pontryagin [2], the concept of roughness was first given, which was later called the concept of roughness in the sense of Andronov–Pontryagin [3].

In the author's papers [12–23], results are obtained that develop the concept of roughness in the sense of Andronov–Pontryagin, allowing one to quantitatively research and solve problems of roughness and bifurcations of dynamical systems which are effectively applied to synergetic systems.

The review considers the development of theories of robustness, roughness and bifurcations of dynamical systems, in particular, in relation to control systems and synergetic systems of various physical natures.

The main stages in the development of the theory of robustness of systems

The traditional understanding of *roughness and robustness* in modern literature defines robustness as the ability of systems to preserve certain properties of not a single system, but a set of systems defined in one way or another under finite parametric or external disturbances, and roughness as a property of systems to preserve a qualitative picture of the partition of the phase space on the trajectory under a small disturbance of the system topologies [2, 3, 15, 24–26].

As mentioned above, the solution of issues of robust stability is primarily associated with the fundamental works of V.L. Kharitonov for interval polynomials [5, 6].

In these works, V.L. Kharitonov solved questions about the stability of interval polynomials (or a family of polynomials) of the form

$$f(\lambda) = b_0\lambda^n + b_1\lambda^{n-1} + \ldots + b_n, \quad (1)$$

where b_i, $i = 0, 1, \ldots, n$ are the coefficients given in the intervals $\underline{b}_i \le b_i \le \overline{b}_i$, $\underline{b}_i$, $\overline{b}_i$ are the lower and upper bounds of the coefficients b_i respectively.

It is shown that the necessary and sufficient conditions for the robust stability of the entire family of real and complex polynomials (1) are, respectively, the stability of four and eight (paired) angular polynomials. These corner polynomials are now called *Kharitonov polynomials*.

At present, the following have been obtained: the edge theorem and discrete analogues and variants of Kharitonov's theorems, frequency criteria for robust stability of the Mikhailov, Nyquist, D-decomposition types [4, 7].

It should be noted that the issues of designing robust nonlinear control systems have not yet been sufficiently considered, especially when the models and parameters of disturbances are uncertain [27, 28].

The works [29–32] present surveys and formulations of robust stability problems which were based on the work of V.L. Kharitonov [5].

In the work of B.T. Polyak, P.S. Shcherbakov [31] the concept of superstability of linear control systems has been proposed. At the same time, superstable systems have convexity properties that allow simple solutions for many classical problems of control theory, in particular, the problem of robust stabilization under matrix uncertainty. But a significant limitation of such systems is the practical narrowness of their class, determined by the conditions for the presence of dominant diagonal elements of the system matrix with negative values.

In the work of V.M. Kuntsevich [32] interesting results on robust stability for linear discrete systems have been obtained. In this case, the matrix of the system is given in the Frobenius form which also narrows the class of considered real systems.

In the works [33, 34], B.R. Barmish and others proposed counterexamples to Bialas' theorem [35], which were annulled in [8].

In the works [36, 37], M. Mansour and others obtained discrete analogs of Kharitonov's weak and strong theorems [5], which have restrictions imposed on the interval domains of the coefficients, or in cases of applying [25] a complex procedure for projecting polynomial roots onto the segment [–1, 1].

In the modern theory of interval dynamical systems, there are two alternative directions [4–7, 10, 27–30, 38]:

1) algebraic or Kharitonian direction;
2) frequency or Tsypkin–Polyak direction.

The author's papers [8–10, 21, 39–42] present original results obtained in the Kharitonian direction for continuous and discrete linear interval dynamical systems which are generally called *the algebraic method of robust stability*. The main results of the method are presented in [10].

The novelty and distinctive feature of the method lies in the fact that for an interval dynamical system with a matrix of general form both in continuous time

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t),\ \mathbf{x}(t_0) = \mathbf{x}_0, \quad (2)$$

and in the discrete case

$$\mathbf{x}(m+1) = \mathbf{A}\mathbf{x}(m),\ m = 1, 2, 3, \ldots, \quad (3)$$

where $\mathbf{x} = \mathbf{x}(t) \in R^n$, $\mathbf{x}(m)$ are state vectors; $\mathbf{A} \in R^{n\times n}$ is an interval matrix with elements a_{ij}, $i, j = \overline{1, n}$, representing interval values $a_{ij} \in [\underline{a}_{ij}, \overline{a}_{ij}]$ with angular values $\underline{a}_{ij}$, $\overline{a}_{ij}$, $\underline{a}_{ij} \le \overline{a}_{ij}$, the necessary and sufficient conditions for the robust stability of systems (2) and (3).

In the continuous case, the so-called successive separate slope coefficients b_i of the characteristic polynomials (1) of system (2) are determined, which are found by optimization methods of nonlinear programming in terms of the interval elements of the matrix $\mathbf{A}$.

In the discrete case, the concepts of points and intervals of intermittency are introduced for the coefficients of the characteristic polynomial of the system [9, 11, 40–43] on the basis of which an algorithm for determining the robust stability of system (3) is formulated.

The interval characteristic polynomial of a discrete system, obtained using the z-transform, has the form

$$f(z) = \det(z\mathbf{I} - \mathbf{A}) = \sum_{i=0}^{n} b_i z^{n-i},\ b_i \in [\underline{b}_i, \overline{b}_i],\ \underline{b}_i \le \overline{b}_i,$$

where $\mathbf{I}$ the identity matrix.

The algebraic method of robust stability, which was developed for linear interval dynamical systems, can be applied *to the research of nonlinear interval systems* [44], based on the use of the provisions of the topological roughness method described in [20]. In this case, the dynamics of systems is considered near its singular trajectories.

In the frequency direction of analysis and synthesis of robust multidimensional systems, a new method of frequency-robust systems [11] is proposed, based on the concepts of modal control.

Structural redundancy of multidimensional objects allows us to set the problem of generalized modal control [11] when solving the problem of synthesizing the control law that delivers to the matrix **F** of the system obtained by aggregating the original multidimensional continuous control object

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu},\ \mathbf{y} = \mathbf{Cx}$$

and the control law implemented as a linear composition of feedforward on exogenous impact and feedback on the state

$$\mathbf{u} = \mathbf{K_g g} - \mathbf{Kx},$$

$$\mathbf{F} = \mathbf{A} - \mathbf{BK},\ \mathbf{G} = \mathbf{BK_g},$$

where $\mathbf{A} \in \mathbf{R}^{n\times n}$, $\mathbf{B}, \mathbf{K} \in \mathbf{R}^{n\times m}$, $\mathbf{K_g} \in \mathbf{R}^{n\times m}$, **u** is the control vector, **g** is the external input vector, n is the order of the control object, are the desired algebraic spectrum of eigenvalues λ, $\sigma\{\mathbf{F}\} = \{\lambda_i, i = \overline{1, n}\}$ and geometric spectrum of eigenvectors ξ, $\{\xi_i: \mathbf{F}\xi_i = \lambda_i\xi_i, i = \overline{1, l, l \le n}\}$, l is the order of the closed system, the feedback matrix **K** on the state of the object is calculated using the relation $\mathbf{K} = \mathbf{HM}^{-1}$, and the matrix **M** is the solution of the Sylvester equation

$$\mathbf{M\Gamma} - \mathbf{AM} = -\mathbf{BH},$$

where $\mathbf{\Gamma} \in \mathbf{R}^{n\times n}$ is the state matrix of the modal model that determines the desired spectrum of the system modes, **M** is the transformation matrix of the bases of similar matrices **Γ** and $\mathbf{F} = \mathbf{A} - \mathbf{BK}$, $\mathbf{H} \in \mathbf{R}^{r\times n}$ is an arbitrary matrix forming an observable pair (**Γ**, **H**) with **Γ**.

The problem of synthesizing multidimensional frequency-robust continuous systems in the class of well-conditioned “input-output” relations can be solved by methods of generalized modal control that provides the system state matrix with a modal-robust representation. At the same time, due to asymptotic properties, the estimate of the frequency condition number of the “input-output” relation in the entire frequency range of exogenous harmonic influence takes on a minimum value, the degree of deviation of which from unity is determined by the degree of deviation from unity of the condition number of the matrix of eigenvectors.

The main stages in the development of the theory of roughness of systems

In modern science, more and more attention is paid to the *roughness* of dynamic systems, and this is primarily due to the increased interest of researchers in the unifying areas of science which include the science of self-developing systems, and phenomena is synergetics. Also important for science is the problem of studying chaotic phenomena or chaos in synergetic systems which are also associated with the problem of the roughness of such systems [45–47].

Synergetics is increasingly intruding into many areas of modern science, both in the natural sciences and in the humanities and social sciences [48–52], in the study of which issues of roughness and bifurcations are of great importance.

In the classical formulation, the questions of *roughness and bifurcations* of dynamical systems were posed by the great French scientist H. Poincaré [1].

Many fundamental results in the theory of roughness and bifurcations were obtained by A.A. Andronov and his school [2, 3].

In the theory of dynamical systems, two different approaches to the roughness problem are known:
- based on the concept of roughness according to Peixoto or otherwise “structural stability” [53];
- on the basis of the Andronov–Pontryagin concept of roughness, when in contrast to the Peixoto concept, ε-closeness of the original and disturbed homeomorphisms is required.

In [12–23, 54], results were obtained that develop the concept of Andronov–Pontryagin roughness, which form the basis of *the topological roughness method* effectively applied to synergetic systems of various physical nature.

The fundamentals of the topological roughness method are given in [20].

In this case, the method is based on the concept of Andronov–Pontryagin roughness when the initial system of the n-th order is considered

$$\dot{\mathbf{z}}(t) = \mathbf{F}(\mathbf{z}(t)), \tag{4}$$

where $\mathbf{z}(t) \in R^n$ is the vector of phase coordinates; **F** is an n-dimensional differentiable vector function.

System (4) is called topologically rough in the sense of Andronov–Pontryagin in some domain G if the original system and the perturbed system defined in the subdomain $\tilde{G}$ of the domain G:

$$\dot{\tilde{\mathbf{z}}} = \mathbf{F}(\tilde{\mathbf{z}}) + \mathbf{f}(\tilde{\mathbf{z}}), \tag{5}$$

are ε-identical in the topological sense.

Systems (4) and (5) are ε-identical if there are open domains D, $\tilde{D}$ in the n-dimensional phase space for $D \subset \tilde{D} \subset \tilde{G} \subset G$:

$$\exists\varepsilon, \delta > 0, \text{ such that, if } \|\mathbf{f}(\tilde{\mathbf{z}})\| < \delta,\ |d\mathbf{f}_i(\tilde{\mathbf{z}})/d\tilde{\mathbf{z}}_j| < \delta,$$
$$i, j = \overline{1, n}, \text{ then } |\,\|\mathbf{z}\| - \|\tilde{\mathbf{z}}\|\,| < \varepsilon, \tag{6}$$
$$\text{or } (\tilde{D}, (2)) \overset{\varepsilon}{\equiv} (D, (1)),$$

otherwise, the partitions of the domains $\tilde{D}$ and D by the trajectories of systems (5) and (4) are ε-identical (they have the same topological structures with trajectories close to ε), where ε and δ are arbitrary small numbers.

If (6) is not satisfied, then system (4) is non-rough in the sense of Andronov–Pontryagin.

The foundations of the theory and method of topological roughness are laid down in [13] where the main definitions are introduced and basic theorems are proved on the necessary and sufficient conditions for roughness near singular points, on the conditions for the existence of a control that delivers roughness to the system, and on the conditions for the occurrence of topology bifurcations in the system [20]. In this case, the measure of roughness is the condition number C of the matrix of reduction to the diagonal (quasi-diagonal) basis of the matrix of the system, in special trajectories (points, lines, manifolds) of the system. The method of topological roughness is a method of quantitative research of the roughness of dynamical systems based on the qualitative concept of roughness according to Andronov–Pontryagin.

The method has been tested in the research of many well-known synergetic systems of various physical nature, such as Lorenz, Rössler, Belousov–Zhabotinsky systems, "predator-prey", "predator-prey-food", Chua chain, Rikitake dynamo, Henon mapping, Hopf bifurcations, models of economic systems such as Schumpeter, Kaldor, etc. [15–23]. At the same time, the results of the method obtained on the above systems are consistent with the known results of other researchers of these systems.

The system (attractor) of Lorenz, which is the most research by many authors due to the fact that this system, is essentially the historically first system (1963) where the validity of the hypothesis of the great French scientist H. Poincaré (1892) on the existence of chaotic motions in a deterministic system.

Researches of the Lorenz system using the measure of roughness C confirmed the main bifurcations of this system [15, 19] described in the literature and meeting the conditions of the criteria given in [20].

The theory and method of topological roughness of systems suppose the formalization of the mathematical model of the systems under study.

In works [19–23], in the case of systems that are weakly formalized and not formalized by mathematical models, the following approach is proposed, namely, **the approach of analogies** of set-theoretic topology and the abstract method to the researches of such systems.

Basic provisions of the analogy approach. Let some set M_1 be given, with which the set M_2 is associated, the relations of which are determined by some *morphism* →, i.e., the relation

$$M_1 \to M_2, \tag{7}$$

such that

$$\mathbf{F}(M_1) = M_2, \tag{8}$$

where **F** is *a functor* serving as a mapping between sets.

Definition 1. Relation (7) defines a certain space of sets $\{M\}$, in which the topology of this space T is defined.

Definition 2. Singular manifolds μ of the space $\{M\}$ are called singular points, singular lines, and multidimensional manifolds in this space, where certain singular (singular) discontinuities are possible in relation (8), in the sense of the topology of T.

Definition 3. *A disturbance* of a set M is a set $\mathbf{F}(M)$ such that $M + \mathbf{F}(M)$ forms *a disturbed set* in the space $\{M\}$.

Definition 4. We introduce a metric δ for disturbance and a metric ε for disturbed sets.

Definition 5. We call the topology of the space $\{M\}$ near some singular manifold μ rough if, under a small disturbance δ of the set M, the disturbed set $M + \mathbf{F}(M)$ differs from the set M by no more than a small ε.

With the definitions introduced above, it is possible to use all the main provisions of the theory and method of the topological roughness of dynamical systems, i.e., consider the issues of maximum roughness and minimum non-roughness, etc.

The main stages in the development of the theory of bifurcations of systems

The concept of the bifurcation of these systems is closely related to the concept of roughness of dynamical systems. As noted earlier, the term *bifurcation* refers to any abrupt change that occurs when changing parameters in any system, whether it be dynamic, ecological, economic, synergetic, etc.

The beginning of work on the theory of bifurcations should be attributed to the works of H. Poincaré where he researches the dependence of equilibrium states on a parameter [1]. The American scientist E. Hopf also made a significant contribution to the theory of bifurcations [55–58].

A.A. Andronov and his school made a huge contribution to the theory of bifurcations [2, 3, 59, 60]. In essence, they considered all questions of bifurcations on the phase plane: bifurcations of equilibrium positions (singular points), bifurcations of limit cycles, etc.

Much attention is paid to the issues of bifurcations in the works of V.I. Arnold, D.V. Anosov and their colleagues [2, 61]. In these works, researches of bifurcations and singularities for large orders of systems ($n \geq 3$) are already being carried out, based on modern topological methods.

The method of topological roughness described above makes it possible to research bifurcations of high-order dynamical systems, in particular, in determining the bifurcations and chaos of synergetic systems using matrix condition numbers [19–23].

Let us present a basic theorem obtained on the basis of the topological roughness method for researches bifurcations of dynamical systems [15, pp. 48–50].

Theorem. In order for a bifurcation of the topological structure to arise in the domain G of a multidimensional dynamical system with the value of the parameter $\mathbf{q} = \mathbf{q}^*$, $\mathbf{q} \in \mathbf{R}^p$, it is necessary and sufficient that:

1) or in the domain G under consideration there exist non-hyperbolic singular points, or orbitally unstable limit cycles, for which:

$$C\{\mathbf{M}(\mathbf{q}^*)\} = \min \sum_{i=1}^{p} C_i\{\mathbf{M}(\mathbf{q})\},$$

where p is the number of common points or limit cycles in the domain G, C_i is condition number of a matrix at the i-th singular point;

2) either in the domain G of the dynamical system there are hyperbolic points or limit cycles for which the following condition is satisfied:

$$C\{\mathbf{M}(\mathbf{q}^*)\} = \infty.$$

Conclusion

The paper provides a brief overview of the main stages in the development of theories of robustness, roughness, and bifurcations of dynamical systems. A bibliography of the author's main publications is given, in which fundamental results in the field of theories of robustness, roughness and bifurcations of dynamical systems in general and synergetic systems in particular are obtained.

Methods for researching and ensuring the robust stability of interval dynamical systems of both algebraic and frequency directions of robust stability are considered. The main results of the original algebraic method of robust stability for continuous and discrete time as well as the method of analysis and synthesis of frequency-robust multidimensional systems are presented. To research the robustness of nonlinear interval systems, it is proposed to use a combination of the algebraic method of robust stability with the provisions of the method of topological roughness.

The main provisions of the theory and method of topological roughness of dynamical systems are presented based on the concept of roughness according to Andronov–Pontryagin and allowing one to quantitatively research the roughness and bifurcations of systems, in particular, synergetic systems of various physical nature.

The analogy approach proposed above can be used for such weakly formalized and non-formalized systems as information systems, social and political systems. At the same time, the main difficulty in studying such systems will be in determining the corresponding sets M, functors **F**, special varieties μ, as well as introducing the metrics δ and ε of the space $\{M\}$, which are problems of perspective.

The main theorem for determining the bifurcations of dynamical systems of any order, obtained on the basis of the method of topological roughness of systems in terms of sets of singular points and limit cycles, is presented. The method was used to study the roughness and bifurcations of synergetic systems and the chaos of such systems as the Lorenz system and attractors of Rossler, Belousov-Zhabotinsky, Chua systems, "predator-prey" and "predator-prey-food", Hopf bifurcations, economic systems of Schumpeter and Kaldor, the Hénon mapping, etc.

References

1. Poincaré H. Mémoire sur les courbes définies par une equation différentielle. *Journal de Mathématiques Pures et Appliquées*. 1881-1886. (in French)
2. Andronov A.A., Pontriagin L.S. Structurally stable systems. *Doklady AN SSSR*, 1937, vol. 14, no. 5, pp. 247–250. (in Russian)
3. Anosov D.V. Structurally stable systems. *Proceedings of the Steklov Institute of Mathematics*, 1986, vol. 169, pp. 61–95.
4. Poliak B.T., Tcypkin Ya.Z. Robust stability of linear systems. Itogi nauki i tehniki. *Tehnicheskaja kibernetika*, 1991, vol. 32, pp. 3–31. (in Russian)
5. Kharitonov V.L. The asymptotic stability of the equilibrium state of a family of systems of linear differential equations. *Differential Equations*, 1978, vol. 14, no. 11, pp. 2086–2088. (in Russian)
6. Kharitonov V.L. On a generalization of a stability criterion. *Izvestija AN Kazahskoj SSR . Serija fiziko-matematicheskaja*, 1978, no. 1, pp. 53–57. (in Russian)
7. Neimark Y.I. Robust stability and D-partition. *Automation and Remote Control*, 1992, vol. 53, no. 7, pp. 957–965.
8. Omorov R.O. Robustness of interval dynamic systems. I. Robustness in continuous linear interval dynamic systems. *Journal of Computer and Systems Sciences International*, 1996, vol. 34, no. 3, pp. 69–74
9. Omorov R.O. Robustness of interval dynamical systems. II. Robustness of discrete linear interval dynamical systems. *Journal of Computer and Systems Sciences International*, 1996, vol. 34, no. 4, pp. 1–5.
10. Omorov R.O. Robustness research of interval dynamic systems by algebraic method. *Scientific and Technical Journal of Information Technologies, Mechanics and Optics*, 2020, vol. 20, no. 3, pp. 364–370. (in Russian). https://doi.org/10.17586/2226-1494-2020-20-3-364-370
11. Ushakov A.V., Akunova A., Omorov R.O., Akunov T.A. *Robust Multidimensional Control Systems: Frequency and Algebraic Methods*. Ed. by R.O. Omorov. Bishkek, Ilim Publ., 2022, 352 p. (in Russian).
12. Omorov R.O. Estimation of roughness of controllable dynamic systems. *Russian Electromechanics*, 1990, no. 7, pp. 81–87. (in Russian)
13. Omorov R.O. Maximal robustness of dynamical systems. *Automation and Remote Control*, 1991, vol. 52, no. 8, pp. 1061–1068.
14. Omorov R.O. Measure of roughness of dynamic systems and criteria of emergence of chaotic fluctuations and bifurcations in synergetic systems. *Synthesis of algorithms of systems stabilization: department proceedings. Issue 8*. Taganrog, 1992, pp. 128–134. (in Russian)
15. Omorov R.O. *Dynamical System Quantitative Robustness Measures and Their Applications to Control Systems*. Dissertation for the degree of doctor of technical sciences, St. Petersburg, Saint Petersburg Institute of Fine Mechanics and Optics, 1992, 188 p. (in Russian)
16. Omorov R.O. Synergetic systems: problems of roughness, bifurcations and accidents. *Nauka i novye tehnologii*, 1997, no. 2, pp. 26–36. (in Russian)

Литература

1. Пуанкаре А. О кривых, определяемых дифференциальными уравнениями / пер. с франц. под ред. А.А. Андронова. М.–Л.: Гостехиздат, 1947. 392 с.
2. Андронов А.А., Понтрягин Л.С. Грубые системы // Доклады АН СССР. 1937. Т. 14. № 5. С. 247–250.
3. Аносов Д.В. Грубые системы // Труды Математического института им. В.А. Стеклова Академии наук СССР. 1985. Т. 169. С. 59–93.
4. Поляк Б.Т., Цыпкин Я.З. Робастная устойчивость линейных систем // Итоги науки и техники. Сер. Техническая кибернетика. ВИНИТИ. 1991. Т. 32. С. 3–31.
5. Харитонов В.Л. Об асимптотической устойчивости положения равновесия семейства систем линейных дифференциальных уравнений // Дифференциальные уравнения. 1978. Т. 14. № 11. С. 2086–2088.
6. Харитонов В.Л. Об одном обобщении критерия устойчивости // Известия АН Казахской ССР. Серия физико-математическая. 1978. № 1. С. 53–57.
7. Неймарк Ю.И. Робастная устойчивость и D-разбиение // Автоматика и телемеханика. 1992. № 7. С. 10–18.
8. Оморов Р.О. Робастность интервальных динамических систем. I. Робастность непрерывных линейных интервальных динамических систем // Известия РАН. Теория и системы управления. 1995. № 1. С. 22–27.
9. Оморов Р.О. Робастность интервальных динамических систем. II. Робастность дискретных линейных интервальных динамических систем // Известия РАН. Теория и системы управления. 1995. № 3. С. 3–7.
10. Оморов Р.О. Алгебраический метод исследования робастности интервальных динамических систем // Научно-технический вестник информационных технологий, механики и оптики. 2020. Т. 20. № 3. С. 364–370. https://doi.org/10.17586/2226-1494-2020-20-3-364-370
11. Ушаков А.В., Акунова А., Оморов Р.О., Акунов Т.А. Робастные многомерные системы управления: Частотные и алгебраические методы / под ред. Р.О. Оморова. Бишкек: Илим, 2022. 352 с.
12. Оморов Р.О. Оценка грубости управляемых динамических систем // Известия вузов. Электромеханика. 1990. № 7. С. 81–87.
13. Оморов Р.О. Максимальная грубость динамических систем // Автоматика и телемеханика. 1991. № 8. С. 36–45.
14. Оморов Р.О. Мера грубости динамических систем и критерии возникновения хаотических колебаний и бифуркаций в синергетических системах // Синтез алгоритмов стабилизации систем: межведомственный сборник. Вып. 8. Таганрог, 1992. С. 128–134.
15. Оморов Р.О. Количественные меры грубости динамических систем и их приложения к системам управления: диссертация на соискание ученой степени доктора технических наук. СПб.: Санкт-Петербургский институт точной механики и оптики, 1992. 188 с.
16. Оморов Р.О. Синергетические системы: проблемы грубости, бифуркаций и катастроф // Наука и новые технологии. 1997. № 2. С. 26–36.

17. Omorov R.O. Method of topological roughness: theory and appendices. I. Theory. *Izvestija Nacional'noj Akademii nauk Kyrgyzskoj Respubliki*, 2009, no. 3, pp. 144–148. (in Russian)
18. Omorov R.O. Topological roughness of synergetic systems. *Journal of Automation and Information Sciences*, 2012, vol. 44, no. 4, pp. 61–70. https://doi.org/10.1615/JAutomatInfScien.v44.i4.70
19. Omorov R.O. *Theory of Topological Roughness of Systems. Applications to Synergetic Systems and Chaos*. Bishkek, Ilim Publ., 2019, 288 p. (in Russian)
20. Omorov R.O. Method of topological roughness of dynamic systems: applications to synergetic systems. *Scientific and Technical Journal of Information Technologies, Mechanics and Optics*, 2020, vol. 20, no. 2, pp. 257–262. (in Russian). https://doi.org/10.17586/2226-1494-2020-20-2-257-262
21. Omorov R.O. *Sensitivity, Robustness and Roughness of Dynamic Systems*. Moscow, LENAND Publ., 2021, 304 p. (in Russian)
22. Omorov R.O. *Synergetics and Chaos: Topological Roughness and Bifurcations*. Moscow, LENAND Publ., 2022, 160 p. (in Russian)
23. Omorov R.O. The modal sensitivity, robustness and roughness of dynamic systems (review article). *Scientific and Technical Journal of Information Technologies, Mechanics and Optics*, 2021, vol. 21, no. 2, pp. 179–190. (in Russian). https://doi.org/10.17586/2226-1494-2021-21-2-179-190
24. Dorato P.D. A historical review of robust control. *IEEE Control Systems Magazine*, 1987, vol. 7, no. 2, pp. 44–47. https://doi.org/10.1109/MCS.1987.1105273
25. Jury E.I. Robustness of a discrete system. *Automation and Remote Control*, 1990, vol. 51, no. 5, pp. 571–592.
26. Discussion on robustness problem in control systems. *Avtomatika I Telemkhanika*, 1992, no. 1, pp. 165–176. (in Russian)
27. Nikiforov V.O. Robust output control for a linear object. *Automation and Remote Control*, 1998, vol. 59, no. 9, pp. 1274–1283.
28. Pelevin A.Ye. Robust control law synthesis under uncertainty of model parameters. *Giroskopiya i Navigatsiya*, 1999, no. 2(25), pp. 63–74. (in Russian)
29. Gusev Yu.M., Yefanov V.N., Krymskiy V.G., Rutkovskiy V.Yu. Analysis and synthesis of linear interval dynamic systems (the state of the problem). I. Analysis which uses interval characteristic polynomials. *Soviet journal of computer and systems sciences*, 1991, vol. 29, no. 6, pp. 84–103
30. Gusev Yu.M., Yefanov V.N., Krymskiy V.G., Rutkovskiy V.Yu Analysis and synthesis of linear interval dynamical systems (the state of the problem). II. Analysis of the stability of interval matrices and synthesis of robust regulators. *Soviet journal of computer and systems sciences*, 1992, vol. 30, no. 2, pp. 26–52
31. Polyak B.T., Shcherbakov P.S. Superstable linear control systems. I. Analysis. *Automation and Remote Control*, 2002, vol. 63, no. 8, pp. 1239–1254. https://doi.org/10.1023/A:1019823208592
32. Kuntsevich V.M. *Management under Conditions of Uncertainty: Guaranteed Results in Management and Identification Issues*. Kiev, Naukova dumka Publ., 2006, 264 p. (in Russian)
33. Barmish B.R., Hollot C.V. Counter-example to a recent result on the stability of interval matrices by S. Bialas. *International Journal of Control*, 1984, vol. 39, no. 5, pp. 1103–1104. https://doi.org/10.1080/00207178408933235
34. Barmish B.R., Fu M., Saleh S. Stability of a polytope of matrices: Counterexamples. *IEEE Transactions on Automatic Control*, 1988, vol. 33, no. 6, pp. 569–572. https://doi.org/10.1109/9.1254
35. Bialas S. A necessary and sufficient condition for the stability of interval matrices. *International Journal of Control*, 1983, vol. 37, no. 4, pp. 717–722. https://doi.org/10.1080/00207178308933004
36. Kraus F.J., Anderson B.D.O., Jury E.I., Mansour M. On the robustness of low-order Schur polynomials. *IEEE Transactions on Circuits and Systems*, 1988, vol. 35, no. 5, pp. 570–577. https://doi.org/10.1109/31.1786
37. Mansour M., Kraus F.J. *On Robust Stability of Schur Polynomials*. Report N 87-05, Inst. Autom. Cont. Ind. Electronics, Swiss, Fed. Inst. Tech. (ETH), Züric, 1987, 34 p.
38. Akunov T.A., Slita O.V., Ushakov A.V. Gramians in parametric invariance of continuous systems. *Scientific and Technical Journal of Information Technologies, Mechanics and Optics*, 2005, vol. 5, no. 3, pp. 39–43. (in Russian)
39. Omorov R.O. Robustness of interval dynamic systems I. Robustness in continuous linear interval dynamic systems. *Journal of Computer and Systems Sciences International*, 1996, vol. 34, no. 3, pp. 69–74.

17. Оморов Р.О. Метод топологической грубости: Теория и приложения. I. Теория // Известия Национальной Академии наук Кыргызской Республики. 2009. № 3. С. 144–148.
18. Omorov R.O. Topological roughness of synergetic systems // Journal of Automation and Information Sciences. 2012. V. 44. N 4. P. 61–70. https://doi.org/10.1615/JAutomatInfScien.v44.i4.70
19. Оморов Р.О. Теория топологической грубости систем. Приложения к синергетическим системам и хаосу. Бишкек: Илим, 2019. 288 с.
20. Оморов Р.О. Метод топологической грубости динамических систем: Приложения к синергетическим системам // Научно-технический вестник информационных технологий, механики и оптики. 2020. Т. 20. № 2. С. 257–262. https://doi.org/10.17586/2226-1494-2020-20-2-257-262
21. Оморов Р.О. Чувствительность, робастность и грубость динамических систем. М.: ЛЕНАНД, 2021. 304 с.
22. Оморов Р.О. Синергетика и хаос: Топологическая грубость и бифуркации. М.: ЛЕНАНД, 2022. 160 с.
23. Оморов Р.О. Модальная чувствительность, робастность и грубость динамических систем (обзорная статья) // Научно-технический вестник информационных технологий, механики и оптики. 2021. Т. 21. № 2. С. 179–190. https://doi.org/10.17586/2226-1494-2021-21-2-179-190
24. Dorato P.D. A historical review of robust control // IEEE Control Systems Magazine. 1987. V. 7. N 2. P. 44–47. https://doi.org/10.1109/MCS.1987.1105273
25. Джури Э.И. Робастность дискретных систем // Автоматика и телемеханика. 1990. № 5. С. 3–28.
26. Дискуссия по проблеме робастности в системах управления // Автоматика и телемеханика. 1992. № 1. С. 165–176.
27. Никифоров В.О. Робастное управление линейным объектом по выходу // Автоматика и телемеханика. 1998. № 9. С. 87–99.
28. Пелевин А.Е. Синтез робастного закона управления при неопределенностях параметров модели объекта // Гироскопия и навигация. 1999. № 2(25). С. 63–74.
29. Гусев Ю.М., Ефанов В.Н., Крымский В.Г., Рутковский В.Ю. Анализ и синтез линейных интервальных динамических систем (состояние проблемы). I. Анализ с использованием интервальных характеристических полиномов // Известия АН СССР. Техническая кибернетика. 1991. № 1. С. 3–23.
30. Гусев Ю.М., Ефанов В.Н., Крымский В.Г., Рутковский В.Ю. Анализ и синтез линейных интервальных динамических систем (состояние проблемы). II. Анализ устойчивости интервальных матриц и синтез робастных регуляторов // Известия АН СССР. Техническая кибернетика. 1991. № 2. С. 3–30.
31. Поляк Б.Т., Щербаков П.С. Сверхустойчивые линейные системы управления. I. Анализ // Автоматика и телемеханика. 2002. № 8. С. 37–53.
32. Кунцевич В.М. Управление в условиях неопределенности: гарантированные результаты в задачах управления и идентификации. Киев: Наукова думка, 2006. 264 с.
33. Barmish B.R., Hollot C.V. Counter-example to a recent result on the stability of interval matrices by S. Bialas // International Journal of Control. 1984. V. 39. N 5. P. 1103–1104. https://doi.org/10.1080/00207178408933235
34. Barmish B.R., Fu M., Saleh S. Stability of a polytope of matrices: Counterexamples // IEEE Transactions on Automatic Control. 1988. V. 33. N 6. P. 569–572. https://doi.org/10.1109/9.1254
35. Bialas S. A necessary and sufficient condition for the stability of interval matrices // International Journal of Control. 1983. V. 37. N 4. P. 717–722. https://doi.org/10.1080/00207178308933004
36. Kraus F.J., Anderson B.D.O., Jury E.I., Mansour M. On the robustness of low-order Schur polynomials // IEEE Transactions on Circuits and Systems. 1988. V. 35. N 5. P. 570–577. https://doi.org/10.1109/31.1786
37. Mansour M., Kraus F.J. On Robust Stability of Schur Polynomials: Report N 87-05, Inst. Autom. Cont. Ind. Electronics, Swiss, Fed. Inst. Tech. (ETH). Züric, 1987. 34 p.
38. Акунов Т.А., Слита О.В., Ушаков А.В. Использование системных грамианов в задачах параметрической инвариантности непрерывных систем // Научно-технический вестник Санкт-Петербургского государственного университета информационных технологий, механики и оптики. 2005. Т. 5. № 3. С. 39–43.
39. Omorov R.O. Robustness of interval dynamic systems I. Robustness in continuous linear interval dynamic systems // Journal of Computer and Systems Sciences International. 1996. V. 34. N 3. P. 69–74.

40. Omorov R.O. Robustness of interval dynamical systems. II. Robustness of discrete linear interval dynamical systems. *Journal of Computer and Systems Sciences International*, 1996, vol. 34, no. 4, pp. 1–5.
41. Omorov R.O. On discrete analogue of Kharitonov's theorem. *Science and New Technologies*, 2002, no. 3, pp. 5–10. (in Russian)
42. Omorov R.O. *Robust Stability of Interval Dynamic Systems*. Bishkek, Ilim Publ., 2018, 104 p. (in Russian)
43. Omorov R.O. Robustness of Interval Dynamic Systems. II. *Radiotehnika i jelektronika*, 1995, vol. 40, no. 12, pp. 3–7.
44. Omorov R.O. Robustness of interval dynamic systems. *Proc. of the Conference "Mathematical Control Theory and Its Applications", St.Petersburg, October 7-8*, 2020. St. Petersburg, Concern CSRI Elektropribor, 2020, pp. 333–335. (in Russian)
45. Haken H. *Advanced Synergetics: Instability Hierarchies of Self-Organizing Systems and Devices*. Berlin, Springer-Verlag, 1983.
46. Nikolis G., Prigogine I. *Exploring Complexity: An Introduction*. W.H. Freeman Publ., 1989, 313 p.
47. *Strange Attractors*. Moscow, Mir Publ., 1981, 253 p. (in Russian)
48. Zhang W.-B. *Synergetic Economics: Time and Change in Nonlinear Economics*. Berlin, Springer-Verlag, 1991, 264 p.
49. Kapitca S.P., Kurdiumov S.P., Malinetckii G.G. *Synergetics and Future Forecasts*. Moscow, URSS Publ., 2001, 288 p. (in Russian)
50. Leonov G.A., Kuznetsov N.V., Kudryashova E.V. Tunisia 2011-2014. Bifurcation, revolution, and controlled stabilization. *Vestnik Sankt-Peterburgskogo universiteta. Prikladnaya matematika informatika protsessy upravleniya*, 2016, no. 4, pp. 92–103. (in Russian). https://doi.org/10.21638/11701/spbu10.2016.409
51. Andrievskii B.R., Fradkov A.L. Control of chaos: Methods and applications. I. Methods. *Automation and Remote Control*, 2003, vol. 64, no. 5, pp. 673–713. https://doi.org/10.1023/A:1023684619933
52. Kolesnikov A.A. *Synergetic Control Methods for Complex Systems: System Synthesis Theory*. Moscow, LIBROKOM Publ., 2012, 240 p. (in Russian)
53. Peixoto M.M. On structural stability. *Annals of Mathematics*, 1959, vol. 69, no. 1, pp. 199–222. https://doi.org/10.2307/1970100
54. Omorov R.O. Topological roughness and bifurcations of synergetic systems. *Proc. of the Conference "Mathematical Control Theory and Its Applications"*, St. Petersburg, October 7-8, 2020. St. Petersburg, Concern CSRI Elektropribor, 2020, pp. 28–30. (in Russian)
55. Aleksandrov P.S. Poincaré and topology. *Poincaré H. Selected Works*. Moscow, Nauka Publ., 1972, pp. 808–816. (in Russian)
56. Hopf E. *Abzweigungen einer periodischen Losung von elner statlonaren Losung eines differential systems*. Wissenschaften, Leipzig, 1942, vol. 94, pp. 1–22.
57. Joss G., Joseph D. *Elementary Stability and Bifurcation Theory*. Springer New York, 1980, 286 p.
58. Chow S.N., Hale J.K. *Methods of Bifurcation Theory*. Springer-Verlag, 1982, 515 p.
59. Leontovich E.A., Gordon I.I., Maier A.G., Andronov A.A. *Bifurcation Theory of Dynamical Systems on the Plane*. Moscow, Nauka Publ., 1967, 488 p. (in Russian)
60. Andronov A.A., Vitt A.A., Khaikin S.E. *Oscillation Theory*. Moscow, 1981, 568 p. (in Russian)
61. Arnol'd V.I. Catastrophe theory. *Itogi Nauki i Tekhniki. Seriya "Sovremennye Problemy Matematiki. Fundamental'nye Napravleniya*. V. 5, Moscow, VINITI, 1986, pp. 219–277. (in Russian)

40. Omorov R.O. Robustness of interval dynamical systems. II. Robustness of discrete linear interval dynamical systems // Journal of Computer and Systems Sciences International. 1996. V. 34. N 4. P. 1–5.
41. Оморов Р.О. О дискретном аналоге теоремы Харитонова // Наука и новые технологии. 2002. № 3. С. 5–10.
42. Оморов Р.О. Робастная устойчивость интервальных динамических систем. Бишкек: Илим, 2018. 104 с.
43. Omorov R.O. Robustness of Interval Dynamic Systems. II // Радиотехника и электроника. 1995. Т. 40. № 12. С. 3–7.
44. Оморов Р.О. Робастность интервальных динамических систем // Математическая теория управления и ее приложения (МТУИП-2020): материалы конференции, Санкт-Петербург, 7-8 октября 2020 г. СПб.: АО «Концерн «ЦНИИ «Электроприбор», 2020. С. 333–335.
45. Haken H. Advanced Synergetics: Instability Hierarchies of Self-Organizing Systems and Devices. Berlin, Springer-Verlag, 1983.
46. Nikolis G., Prigogine I. Exploring Complexity: An Introduction. W.H. Freeman Publ., 1989. 313 p.
47. Странные аттракторы: Сб. статей / пер. с англ. под ред. Я.Г. Синая, Л.П. Шильникова. М.: Мир, 1981. 253 с.
48. Zhang W.-B. Synergetic Economics: Time and Change in Nonlinear Economics. Berlin: Springer-Verlag, 1991. 264 p.
49. Капица С.П., Курдюмов С.П., Малинецкий Г.Г. Синергетика и прогнозы будущего. 2-е изд. М.: Эдиториал УРСС, 2001. 288 с.
50. Леонов Г.А., Кузнецов Н.В., Кудряшова Е.В. Тунис 2011-2014. Бифуркация, революция и управляемая стабилизация // Вестник Санкт-Петербургского университета. Прикладная математика. Информатика. Процессы управления. 2016. № 4. С. 92–103. https://doi.org/10.21638/11701/spbu10.2016.409
51. Andrievskii B.R., Fradkov A.L. Control of chaos: Methods and applications. I. Methods // Automation and Remote Control. 2003. V. 64. N 5. P. 673–713. https://doi.org/10.1023/A:1023684619933
52. Колесников А.А. Синергетические методы управления сложными системами: Теория системного синтеза. 2-е изд. М.: Книжный дом «ЛИБРОКОМ», 2012. 240 с.
53. Peixoto M.M. On structural stability // Annals of Mathematics. 1959. V. 69. N 1. P. 199–222. https://doi.org/10.2307/1970100
54. Оморов Р.О. Топологическая грубость и бифуркации синергетических систем // Математическая теория управления и ее приложения (МТУИП-2020): материалы конференции, Санкт-Петербург, 7-8 октября 2020 г. СПб.: АО «Концерн «ЦНИИ «Электроприбор», 2020. С. 28–30.
55. Александров П.С. Пуанкаре и топология (Poincaré and topology) // Пуанкаре А. Избранные труды. Т 2. М.: Наука, 1972. pp. 808–816.
56. Hopf E. Abzweigungen einer periodischen Losung von elner statlonaren Losung eines differential systems. Wissenschaften, Leipzig, 1942. V. 94. P. 1–22.
57. Йосс Ж., Джозеф Д. Элементарная теория устойчивости и бифуркаций. М.: Мир, 1983. 301 с.
58. Chow S.N., Hale J.K. Methods of Bifurcation Theory. Springer-Verlag, 1982. 515 p.
59. Леонтович Е.А., Гордон И.И., Майер А.Г., Андронов А.А. Теория бифуркаций динамических систем на плоскости. М.: Наука, 1967. 488 с.
60. Андронов А.А., Витт А.А., Хайкин С.Э. Теория колебаний. М.: Наука, 1981. 568 с.
61. Арнольд В.И. Теория катастроф // Итоги науки и техники. Серия «Современные проблемы математики. Фундаментальные направления». Т. 5. М.: ВИНИТИ, 1986, С. 219–277.

Author

Roman O. Omorov — D.Sc. (Engineering), Professor, Correspondent member of National Academy of Sciences of the Kyrgyz Republic, Head of the Laboratory, Institute of Machine Science and Automation of the National Academy of Sciences of the Kyrgyz Republic, Bishkek, 720071, Kyrgyz Republic, sc 6602708366, https://orcid.org/0000-0003-3555-1323, romano_ip@list.ru

Автор

Оморов Роман Оморович — доктор технических наук, профессор, член-корреспондент Национальной академии наук Кыргызской Республики, заведующий лабораторией, Институт машиноведения и автоматики Национальной академии наук Кыргызской Республики, Бишкек, 720071, Кыргызская Республика, sc 6602708366, https://orcid.org/0000-0003-3555-1323, romano_ip@list.ru

Received 23.10.2022
Approved after reviewing 05.01.2023
Accepted 30.03.2023

Статья поступила в редакцию 23.10.2022
Одобрена после рецензирования 05.01.2023
Принята к печати 30.03.2023