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Modeling of nonlocal porous functionally graded nanobeams under moving loads

Ridha A. Ahmed[1], Wael N. Abdullah[2], Nadhim M. Faleh[3]✉, Mamoon A. Al-Jaafari[4]

[1,2,3,4] Engineering College of Al-Mustansiriyah University, Bahdad, 10001, Iraq

[1] ridhalwan@uomustansiriyah.edu.iq, https://orcid.org/0000-0002-6666-6564
[2] wael.najm65@gmail.com, https://orcid.org/0000-0001-9599-4279
[3] dr.nadhim@uomustansiriyah.edu.iq✉, https://orcid.org/0000-0003-0003-1643
[4] dr.mamoonal-jaafare@uomustansiriyah.edu.iq, https://orcid.org/0009-0005-9597-9825

Abstract
This study focuses on the dynamic response of porous functionally graded nanomaterials to moving loads. The analysis was performed using two approaches: the Ritz method with the help of the benefits achieved by employing Chebyshev polynomials in the cosine form and the differential quadrature method with further inverse Laplace transformation. Both approaches utilize the formulation of a nano-thin beam considering an improved higher-order beam model and nonlocal strain gradient theory with two characteristic length scales, referred to as nonlocality and strain gradient length scales. Power-law dependencies steer the constituent designs of pore-graded materials toward pore factors that influence pore volume either with a uniform or non-uniform distribution of pores. Moreover, a variable scale modulus was adopted to further improve accuracy by considering the scale effects for graded nano-thin beams. The first part of the study addresses the equation of motion, which is solved by applying the Ritz technique with Chebyshev polynomials. In the second part, the governing equations for nanobeams are discussed where the differential quadrature method is used to discretise them further, and the inverse Laplace transform is used to obtain the dynamic deflections. The results of the present study elucidate the effects of the moving load speed, nonlocal strain gradient factors, porosity, pore number and distribution, and elastic medium on the dynamic deflection of functionally graded nanobeams.

Keywords
design, material, gradient porous, moving load, nonlocal strain, porous

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Моделирование нелокальных пористых функционально-градиентных нанобалок под действием движущихся нагрузок

Ридха А. Ахмед[1], Ваэл Н. Абдулла[2], Надхим М. Фалех[3]✉, Мамун А. Аль-Джаафари[4]

[1,2,3,4] Университет Аль-Мустансирия, Багдад, 10001, Ирак

[1] ridhalwan@uomustansiriyah.edu.iq, https://orcid.org/0000-0002-6666-6564
[2] wael.najm65@gmail.com, https://orcid.org/0000-0001-9599-4279
[3] dr.nadhim@uomustansiriyah.edu.iq✉, https://orcid.org/0000-0003-0003-1643
[4] dr.mamoonal-jaafare@uomustansiriyah.edu.iq, https://orcid.org/0009-0005-9597-9825

Аннотация
Исследован динамический отклик пористых функционально-градиентных наноматериалов на движущиеся нагрузки. Анализ проводился с использованием двух подходов: метода Ритца с использованием преимуществ, достигаемых за счет внедрения полиномов Чебышева в форме косинуса, и метода дифференциальных квадратур с последующим обратным преобразованием Лапласа. Оба подхода применяют модель нанотонкой балки с учетом улучшенной модели более высокого порядка и нелокальной теории градиента деформации с двумя характерными шкалами длины, называемыми шкалами нелокальности и градиента деформации. Степенные

зависимости ориентируются на составных конструкциях пористых материалов с различными факторами: объем пор, равномерное или неравномерное распределение пор. Принят переменный модуль масштаба для дальнейшего повышения точности за счет учета масштабных эффектов для градуированных нанотонких балок. В первой части работы рассмотрено уравнение движения, которое решено путем применения техники Ритца с полиномами Чебышева. Во второй части выполнен анализ уравнения для нанобалок, где применен метод дифференциальных квадратур для их дальнейшей дискретизации, а обратное преобразование Лапласа использовано для получения динамических прогибов. Результаты исследования позволяют объяснить влияние скорости движущейся нагрузки, нелокальных факторов градиента деформации, пористости, числа и распределения пор, а также упругой среды на динамический прогиб функционально-градуированных нанобалок.

Ключевые слова

конструкция, материал, градиентная пористость, подвижная нагрузка, нелокальная деформация, пористость

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Introduction

Mechanical characteristics of nanoscale structures, including nanoscale beams or nanosized plates, based on non-classic elasticity theories have been an important case study in recent decades [1]. This is because of the broad application of nanoscale structures in nanosensors or nanoelectromechanical systems. The most familiar theory for modeling nanoscale structures is nonlocal elasticity [2]. In these theories, scale factors were introduced to interpolate the influence of small sizes. Pursuant to Nonlocal Elasticity Theory (NET), the stress field must be nonlocal because the relationship between the stress and strain at a point depends on the strains at that point and the surrounding points [3]. The nonlocality of the stress field is examined using nonlocal parameters. Any value of a nonlocal parameter can be determined through experiments or numerical simulations. However, deriving the values of these nonlocal parameters using these methods can be complicated and time-consuming. Consequently, considerable academic research on the static and dynamic aspects of nanoscale structures has been conducted as parameter-based studies that rely on specific theoretical values for the nonlocality modulus [3–9].

Recently, several theoretical studies and experiments have reported that small-sized effects must be characterized via stiffness-increasing mechanisms or strain gradient fields [10]. This assertion differs from that of nonlocal elasticity in which stiffness reduction behavior has been stated [11–16]. However, the effects of the reduction and increment on the structural stiffness at the nanoscale can be considered in the context of the nonlocality strain gradient theory. According to the Nonlocal Strain Gradient Theory (NSGT), two criterion factors termed nonlocal and strain gradient factors have been utilized to provide an excellent description of small-size effects. The static and dynamic properties of nanobeams and other nanostructures have been studied extensively using [17, 18]. The impact of different loadings on the vibration behavior of nanobeams has become an essential case study in recent years. Some of these loadings include harmonic forces, impulsive loads, and moving loads from the overhead view of the nanosized beam. Several authors have investigated the forced vibrations of nanobeams owing to harmonic and impulsive loads in the context of the NET and NSGT. However, causing vibrations of nanosized beams owing to mobile loads has become important because of nano-sensing and nano-probing applications [19–26]. It has been realized that the dynamic deflections of a nanobeam owing to moving loads increase with the inclusion of nonlocal parameters [27].

Method of formulation

As discussed, the influences of the reduction and increment in the structural stiffness at the nanoscale can be considered in the context of the NSGT. According to that theory, two scale criteria termed nonlocal and strain gradient factors have been utilized to provide an excellent description of small-size effects. First, it is essential to define the stress-field components as follows:

$$\sigma_j = \sigma_j^{(0)} - \nabla\sigma_j^{(1)},$$

where σ is stress components; $\sigma_j^{(1)}$ and $\sigma_j^{(0)}$ are stress components; ∇ is gradient; $\nabla\sigma_j$ stress gradients.

ε_k and strain gradients $\nabla\varepsilon_k$ can be defined as:

$$\sigma_j^{(0)} = \int_V C_{jk}\,\alpha_0(x, x', q_0 u)\,\varepsilon'_k(x')\,dx'$$

$$\sigma_j^{(1)} = J^2 \int_V C_{jk}\,\alpha_1(x, x', q_1 u)\,\nabla\varepsilon'_k(x')\,dx'$$

where the symbol C_{jk} is used for the elastic coefficients, $q_0 u$ and $q_1 u$ are used to define the nonlocal effects, and J is Strain gradient factor which introduces the influences of the strain gradients. If the nonlocal functions $\alpha_0(x, x', q_0 u)$ and $\alpha_1(x, x', q_1 u)$ can satisfy the conditions introduced by [28], the relationship between the stresses and strains in the context of NSGT becomes:

$$\begin{gathered}[1 - (q_1 u)^2\nabla^2][1 - (q_0 u)^2\nabla^2]\sigma_j = \\ = C_{jk}[1 - (q_1 u)^2\nabla^2]\varepsilon_k - C_{jk} J^2[1 - (q_0 u)^2\nabla^2]\nabla^2\varepsilon_k,\end{gathered} \tag{1a}$$

where ∇^2 is the Laplacian operator; q is combined nonlocal parameter. J^2 is strain gradient length scale. The above relation can be simplified by assuming $q_1 u = q_0 u = qu$ as:

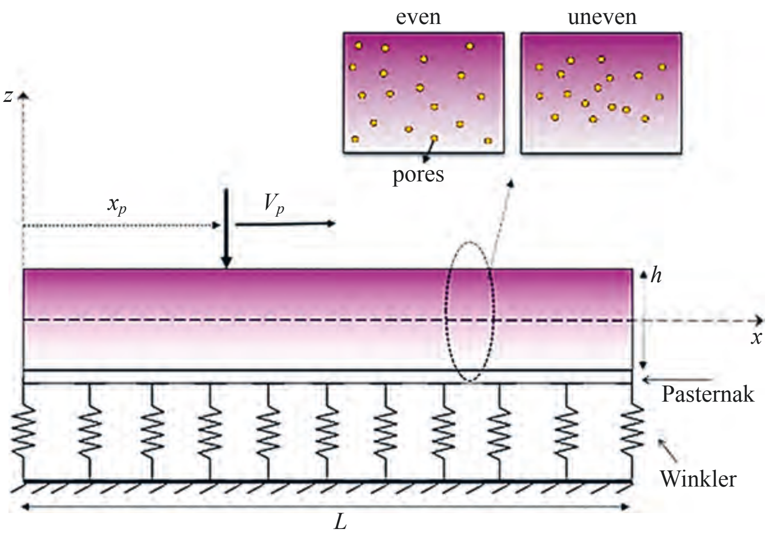

Fig. 1. Gradation nano-thickness beam undersurface of a movable particle

$$[1-(ea)^2\nabla^2]\sigma_{ij} = C_{ijkl}[1-\nabla^2]\varepsilon_{kl}. \quad (1)$$

The distribution of Functionally Graded (FG) materials in structures can be mathematically modeled using the power law or Mori-Tanaka models. Using the power-law function, it is possible to describe the continuous gradation of material properties with good accuracy. However, the Mori-Tanaka scheme provided more accurate results as reported in some studies. In the first step, we assumed an FG nanobeam of length L and thickness h as shown in Fig. 1, where x_p denotes the instantaneous position of the moving load along the beam length, while V_p indicates the constant velocity of that moving load. Fig. 1 illustrates the configuration of the stepped nanobeam under the influence of the moving load, with x_p and V_p corresponding to the load position and velocity, respectively. According to the Mori-Tanaka FG model, the effective properties of the FG nanobeam, including its bulk modulus K_e and shear modulus G_e, are considered, by the following:

$$\frac{K_e - K_m}{K_c - K_m} = \frac{V_c}{1 + V_m(K_c - K_m)/(K_m + 4G_m/3)},$$

$$\frac{G_e - G_m}{G_c - G_m} = \frac{V_c}{1 + V_m(G_c - G_m)/[(G_m + G_m(9K_m + 8G_m)/6(K_m + 2G_m))]}.$$

Here, K_m and K_c denote the bulk modules of the metal and ceramic parts, respectively; G_m and G_c denote the shear modules of the metal and ceramic parts, respectively; V_c is the volume portion of the ceramic constituent; and V_m is the volume portion of the metal constituent. The following relationship with the volume portion of the metal constituent:

$$V_c + V_m = 1,$$

$$V_c(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^p,$$

where h is the thickness of the nanobeam; z is coordinated along the thickness direction and p is the power-law exponent (material gradient index). The distribution of the FG material along the thickness path depends on the value of the material gradient index (p). According to the above relations, one can express the effectual Young's modulus (E), Poisson's ratio (v), and mass density (ρ) of such material as:

$$E(z) = \frac{9K_eG_e}{3K_e + G_e},$$

$$\nu(z) = \frac{3K_e - 2G_e}{6K_e + 2G_e},$$

$$\rho(z) = \rho_c V_c + \rho_m V_m.$$

As mentioned, another approach for modeling a foam gradient material is the power-law function. According to the power-law model, each material property can be defined using the following relation:

$$\Psi(z) = (\Psi_c - \Psi_m)\left(\frac{z}{h} + \frac{1}{2}\right)^p + \Psi_m,$$

where Ψ_m and Ψ_c are the material exclusivity of metallic and ceramic.

Based on the above relationship, it is possible to designate Young's modulus (E), Poisson's ratio (v), and strain gradient factor (J_2). Thus, the effect of graded nonlocality was considered in [29]. However, the above relationships ignore the effect of the porosity inside the FG materials.

Using modified power-law functions, it is possible to model each material property containing porosity volume (ζ) as:

$$\Psi(z) = (\Psi_c - \Psi_m)\left(\frac{z}{h} + \frac{1}{2}\right)^p + \Psi_m - (\Psi_c + \Psi_m)\frac{\zeta}{2}$$

for even distribution,

$$\Psi(z) = (\Psi_c - \Psi_m)\left(\frac{z}{h} + \frac{1}{2}\right)^p + \Psi_m - \frac{\xi}{2}(\Psi_c + \Psi_m)\left(1 - \frac{2|z|}{h}\right)$$

for uneven distribution,

where $\Psi(z)$ is generic material property (e.g., Young's modulus, density). The ξ is a constant with $\xi = \pi/h$. Higher-order beam theories are useful in establishing the governing equations of beams by considering the effects of shear deformation. A well-known theory is the refined beam theory which has the following form of a displacement field (u_x, 0, u_z):

$$u_x(x,z,t) = u(x,t) - (z - s^*)\frac{\partial w_b}{\partial x} - [H(z) - s^{**}]\frac{\partial w_s}{\partial x},$$

$$u_z(x,z,t) = w_b(x,t) + w_s(x,t).$$

The displacement field contains the axial displacement (u), bending displacement (w_b), shear displacement (w_s), s^* is the neutral-axis location parameter, s^{**} is the neutral-axis location parameter, and $H(z)$ is the trigonometric shear strain function. Moreover, to determine the neutral-axis location of the FG beam, it is necessary to calculate

$$s^* = \int_{-0.5h}^{0.5h} E(z)z\,dz \Big/ \int_{-0.5h}^{0.5h} E(z)\,dz,$$

$$s^{**} = \int_{-0.5h}^{0.5h} E(z)H(z)\,dz \Big/ \int_{-0.5h}^{0.5h} E(z)\,dz.$$

The function $H(z)$ has been selected as:

$$H(z) = z - \sin(\xi z)/\xi.$$

The strains derived based on the displacement factors can be expressed as follows:

$$\varepsilon_x = \frac{\partial u}{\partial x} - (z - s^*)\frac{\partial^2 w_b}{\partial x^2},$$

$$\gamma_{xz} = \Omega(z)\frac{\partial w_s}{\partial x}, \text{ where } \Omega(z) = 1 - \frac{dH(z)}{d(z)},$$

ε_x is the normal strain component, and γ_{xz} is the shear strain component.

Note that $\Omega(z) = 1 - dH(z)/dz$. Subsequently, the generalized version of Hamilton's formula implies that:

$$\int_0^t \delta(U - V - K)\,dt = 0. \tag{2}$$

Hitherward, U may be termed the strain energy, V may be termed the work accomplished by external loading, and K may be termed the kinetic energy. The strain energy variation may be composed of

$$\delta U = \int_v \sigma_{ij}\,\delta\varepsilon_{ij}\,dV =$$
$$= \int_v \left(\sigma_x\,\delta\varepsilon_x + \sigma_x^{(1)}\delta\nabla\varepsilon_x + \sigma_{xz}\,\delta\gamma_{xz} + \sigma_{xz}^{(1)}\delta\nabla\gamma_{xz}\right)dV \tag{3}$$

$$\delta U = \int_0^L \left(N\frac{\partial \delta u}{\partial x} - M_b\frac{\partial^2 \delta w_b}{\partial x^2} - M_s\frac{\partial^2 \delta w_s}{\partial x^2} + Q\frac{\partial \delta w_s}{\partial x}\right)dx \tag{4}$$

where N is the resultant force along the beam axial direction; M_b is the moment due to bending stresses; M_s is the moment associated with shear deformation; Q is the transverse force due to shear stresses. Wherein all forces and moments revealed within the overhead equation might be composed as:

$$\begin{aligned}
N_x &= \int_{-h/2}^{h/2}\left(\sigma_x^{(0)} - \nabla\sigma_x^{(1)}\right)dz = N_x^{(0)} - \nabla N_x^{(1)},\\
M_x^b &= \int_{-h/2}^{h/2} z\left(\sigma_x^{(0)} - \nabla\sigma_x^{(1)}\right)dz = M_x^{b(0)} - \nabla M_x^{b(1)},\\
M_x^s &= \int_{-h/2}^{h/2} f\left(\sigma_x^{(0)} - \nabla\sigma_x^{(1)}\right)dz = M_x^{s(0)} - \nabla M_x^{s(1)},\\
Q_{xz} &= \int_{-h/2}^{h/2} g\left(\sigma_{xz}^{(0)} - \nabla\sigma_{xz}^{(1)}\right)dz = Q_{xz}^{(0)} - \nabla Q_{xz}^{(1)},
\end{aligned} \tag{5}$$

where:

$$\begin{aligned}
N_x^{(0)} &= \int_{-h/2}^{h/2}\left(\sigma_x^{(0)}\right)dz, & N_x^{(1)} &= \int_{-h/2}^{h/2}\left(\sigma_x^{(1)}\right)dz\\
M_x^{b(0)} &= \int_{-h/2}^{h/2} z\left(\sigma_x^{(0)}\right)dz, & M_x^{b(1)} &= \int_{-h/2}^{h/2} z\left(\sigma_x^{(1)}\right)dz\\
M_x^{s(0)} &= \int_{-h/2}^{h/2} f\left(\sigma_x^{(0)}\right)dz, & M_x^{s(1)} &= \int_{-h/2}^{h/2} f\left(\sigma_x^{(1)}\right)dz\\
Q_{xz}^{(0)} &= \int_{-h/2}^{h/2} g\left(\sigma_{xz}^{(0)}\right)dz, & Q_{xz}^{(1)} &= \int_{-h/2}^{h/2} g\left(\sigma_{xz}^{(1)}\right)dz
\end{aligned} \tag{6}$$

The variation in the work accomplished via practical loading can be formulated as follows:

$$\delta V = \int_0^L (q\,\delta(w_b + w_s))\,dx \tag{7}$$

The outward lateral loading q might be composed as:

$$q = +k_w(w_b + w_s) - k_p\frac{\partial^2(w_b + w_s)}{\partial x^2} + q_{dyn}. \tag{8}$$

In the above relation, k_w and k_p, respectively, denote the Winkler and Pasternak parts of elastic substrate; $q_{dyn} = P_0\delta(x - V_p t)$ is the applied force owing to the moving load, where V_p denotes the velocity of the moving load, P_0 exhibits the valence of the mobile force and x exhibits the applying status of the mobile force. The variation in kinetic energy is formulated as follows:

$$\begin{aligned}
\delta K = \int_0^L \Bigg(& I_0\left[\frac{\partial u}{\partial t}\frac{\partial \delta u}{\partial t} + \left(\frac{\partial w_b}{\partial t} + \frac{\partial w_s}{\partial t}\right)\left(\frac{\partial \delta w_b}{\partial t} + \frac{\partial \delta w_s}{\partial t}\right)\right] -\\
& - I_1\left(\frac{\partial u}{\partial t}\frac{\partial^2 \delta w_b}{\partial x\partial t} + \frac{\partial^2 w_b}{\partial x\partial t}\frac{\partial \delta u}{\partial t}\right) + I_2\left(\frac{\partial^2 w_b}{\partial x\partial t}\frac{\partial^2 \delta w_b}{\partial x\partial t}\right) -\\
& - J_1\left(\frac{\partial u}{\partial t}\frac{\partial^2 \delta w_s}{\partial x\partial t} + \frac{\partial^2 w_s}{\partial x\partial t}\frac{\partial \delta u}{\partial t}\right) + K_2\left(\frac{\partial^2 w_s}{\partial x\partial t}\frac{\partial^2 \delta w_s}{\partial x\partial t}\right) +\\
& + J_2\left(\frac{\partial^2 w_b}{\partial x\partial t}\frac{\partial^2 \delta w_s}{\partial x\partial t} + \frac{\partial^2 w_s}{\partial x\partial t}\frac{\partial^2 \delta w_b}{\partial x\partial t}\right)\Bigg)dx
\end{aligned} \tag{9}$$

Where,

$$(I_0, I_1, J_1, I_2, J_2, K_2) = \int_{-h/2}^{h/2} (1, z - z^*, (z - z^*)^2, H - z^{**}, (z - z^*)(H - z^{**}), (H - z^{**})^2)\rho(z)dz \tag{10}$$

In Eq. (10): I_0: Mass per unit length of the nanobeam (integral of density $\rho(z)$ over the thickness); I_1: First-order moment coupling axial and bending displacements; J_1: Second-order moment linked to shear deformation; I_2: Second-order moment associated with bending deformation; J_2: Cross-term moment coupling bending and shear deformations; and K_2: Higher-order moment linked to shear deformation. The '1' under the integral represents the unity term contributing to the total mass (I_0) by integrating $\rho(z)$ across the thickness. Finally, the governing equations for a pore-dependent and nano-thickness beam under a movable particle can be obtained by substituting Eqs. (3)–(9) into (2) as.

$$\frac{\partial N}{\partial x} = I_0 \frac{\partial^2 u}{\partial t^2} - I_1 \frac{\partial^3 w_b}{\partial x \partial t^2} - J_1 \frac{\partial^3 w_s}{\partial x \partial t^2},$$

$$\frac{\partial^2 M_b}{\partial x^2} - q_{pth} = + I_0 \left(\frac{\partial^2 w_b}{\partial t^2} + \frac{\partial^2 w_s}{\partial t^2} \right) + I_1 \frac{\partial^3 u}{\partial x \partial t^2} -$$

$$- I_2 \frac{\partial^4 w_b}{\partial x^2 \partial t^2} - J_2 \frac{\partial^4 w_s}{\partial x^2 \partial t^2} + k_w(w_b + w_s) - k_p \frac{\partial^2 (w_b + w_s)}{\partial x^2},$$

$$\frac{\partial^2 M_s}{\partial x^2} + \frac{\partial Q}{\partial x} - q_{pth} = + I_0 \left(\frac{\partial^2 w_b}{\partial t^2} + \frac{\partial^2 w_s}{\partial t^2} \right) +$$

$$+ J_1 \frac{\partial^3 u}{\partial x \partial t^2} - J_2 \frac{\partial^4 w_b}{\partial x^2 \partial t^2} - K_2 \frac{\partial^4 w_s}{\partial x^2 \partial t^2} +$$

$$+ k_w(w_b + w_s) - k_p \frac{\partial^2 (w_b + w_s)}{\partial x^2}.$$

Using Eq. ($1a$) and ($1b$), it is possible to establish the stress-strain relations of a higher-order refined FG nanobeam in the context of the NSGT as

$$\sigma_x - (e_0 a)^2 \frac{\partial^2 \sigma_x}{\partial x^2} = \left(1 - l^2 \frac{\partial^2}{\partial x^2}\right) E(z) \varepsilon_x,$$

$$\tau_{xz} - (e_0 a)^2 \frac{\partial^2 \tau_{xz}}{\partial x^2} = \left(1 - l^2 \frac{\partial^2}{\partial x^2}\right) G(z) \gamma_{xz}.$$

Nano-thickness beam contains the above stresses which result in the following forces and moments by integrating Eqs. (4) and (5) over the thickness. Thus, the internal strength can be determined by:

$$\{A^b, B^b, B_s^b, D^b, D_s^b, H_s^b\} =$$

$$= \int_{-h/2}^{h/2} E(z)\{1, (z - z^*), (H - z^{**}), (z - z^*)^2, (z - z^*)(H - z^{**}), (H - z^{**})^2\} dz$$

$$A_s^b = \int_{-h/2}^{h/2} \Omega^2 G(z) dz$$

$$\{A^{\bar{b}}, B^{\bar{b}}, B_s^{\bar{b}}, D^{\bar{b}}, D_s^{\bar{b}}, H_s^{\bar{b}}\} =$$

$$= \int_{-h/2}^{h/2} E(z) l^2(z)\{1, (z - z^*), (H - z^{**}), (z - z^*)^2, (z - z^*)(H - z^{**}), (H - z^{**})^2\} dz$$

$$A_s^{\bar{b}} = \int_{-h/2}^{h/2} \Omega^2 l^2(z) G(z) dz$$

The dynamic response problem of a nanothick beam is solved in this section basing on the Chebyshev–Ritz method. First, the field components can be assumed to have the following form:

$$u(x,t) = R^u(x) \sum_{m=1}^{\infty} U_m P_m(x) e^{i\omega_n t}, \tag{11}$$

$$w_b(x) = R^{w_b}(x) \sum_{m=1}^{\infty} W_{bm} P_m(x) e^{i\omega_n t}, \tag{12}$$

$$w_s(x) = R^{w_s}(x) \sum_{m=1}^{\infty} W_{sm} P_m(x) e^{i\omega_n t}.$$

Consider the following essential boundary conditions. Simply-supported (SS) edges:

$$w = \frac{\partial^2 w}{\partial x^2} = 0 \text{ at } x = +0.5L, -0.5L$$

In addition, $P_m(x)$ is the n-th Chebyshev polynomial of the 1st type and is expressed as

$$P_m(x) = \cos\left[(m - 1)\arccos\left(\frac{2x}{L}\right)\right].$$

Note that R^i(© = u, w_b, w_s) denotes the functions associated with the essential boundary conditions. In addition, functions can be introduced in the following form:

$$R^i(x) = \left(1 + \frac{2x}{L}\right)^{p^*} \left(1 - \frac{2x}{L}\right)^{q^*},$$

where p^* and q^* are affected by the pattern of the edge conditions. For the SS edges, $p^* = q^* = 1$. Representing Eqs. (10)–(12) in weak form, together with their minimization of the indefinite variables U_m, W_{bm}, and W_{sm}, the equation below results in simultaneous algebraic equations with respect to the unknown coefficients.

$$\frac{\partial \Pi}{\partial U_m} = \frac{\partial \Pi}{\partial W_{bm}} = \frac{\partial \Pi}{\partial W_{sm}} = 0.$$

Substituting Eqs. (11) and (12) into the obtained relation with the knowledge that the axial and transverse motions are uncoupled yields

$$\left\{[K] + \frac{\partial}{\partial t^2}[M]\right\} \begin{Bmatrix} W_b \\ W_s \end{Bmatrix} = \begin{Bmatrix} q_{pth} - \mu_c \dfrac{\partial^2 q_{pth}}{\partial x^2} \\ q_{pth} - \mu_c \dfrac{\partial^2 q_{pth}}{\partial x^2} \end{Bmatrix}, \tag{13}$$

where $[K]$ and $[M]$ are the toughness and mass matrices of the considered nanosized beam, respectively. In addition, the non-dimensional substrate modules can be distinguished by

$$K_w = k_w \frac{L^4}{E_m I},\ K_p = k_p \frac{L^2}{E_m I}.$$

It can be inferred that the dynamic force is mobile along an upstanding path which yields forcing vibrations, and is distinguished by the following formulation:

$$q(x,t) = \sum_{n=1}^{\infty} Q_n \sin\left[\frac{n\pi}{L} x\right],$$

$$Q_n = \frac{2}{L}\int_0^L \sin\left[\frac{n\pi}{L} x\right] q(x)\,dx = \frac{2P_0}{L}\sin\left[\frac{n\pi}{L} x_p\right] = \frac{2P_0}{L}\sin\left[\frac{n\pi}{L} V_p t\right],$$

where Q_n expose the Fourier multipliers and $q(x) = P_0\delta(x - x_p)$; P_0 exhibits the valence of the mobile force and x_p exhibits the applying status of the mobile force. In addition, V_p represents the celerity of the mobile force.

Finally, based on zero primary status and the Laplace transformation branch of knowledge, Eq. (13) changes to

$$\{[\mathbf{K}] + S^2[\mathbf{M}]\}\begin{Bmatrix} [W_b] \\ [W_s] \end{Bmatrix} = \begin{Bmatrix} \left[\mathcal{L}q(x,t) - \mu_c \dfrac{\partial^2 \mathcal{L}q(x,t)}{\partial x^2}\right] \\ \left[\mathcal{L}q(x,t) - \mu_c \dfrac{\partial^2 \mathcal{L}q(x,t)}{\partial x^2}\right] \end{Bmatrix}. \tag{14}$$

By solving Eq. (14) using the inverse Laplace transformation approach, the values of the bending (W_b) and shear (W_s) displacements can be derived. However, the total deflection of the nanobeam is the sum of the two displacements, $W = W_b + W_s$. In addition, the normalized deflection and normalized velocity factors may be introduced by

$$\bar{W} = W\frac{100E_m I}{P_0 L^3},\ V^* = \frac{V_p}{V_{cr}},\ V_{cr} = \frac{\omega_n L}{\pi}.$$

The normalized deflection $\bar{W}$ is introduced to non-dimensionalise the beam dynamic response, enabling comparison across different material and geometric configurations. Similarly, the dimensionless velocity ratio $V^* = V_p/V_{cr}$ characterizes the moving load speed relative to the critical velocity $V_{cr} = \omega_1 L/\pi$, where ω_1 is the fundamental natural frequency of the nanobeam. The critical velocity V_{cr} represents the threshold speed at which the dynamic deflection peaks before sharply decreasing, as observed in Fig. 3. Finally, the normalized time t^* can be expressed as follows:

$$t^* = \frac{V_p t}{L}.$$

Main results

The present section examines the dynamic responses of porous FG nanobeams to moving loads that capture both nonlocality and strain gradient influences. The effects of the load velocity, material gradation, porosity distribution, graded nonlocality, and two-scale factors on the dynamic deflection of the nanobeam were studied in detail. In this study, it was determined that the nonlocality and strain gradient multipliers were not constant for the FG nanobeam. They are variable in the direction of thickness as $\mu(z)$ and $\lambda(z)$ according to Eq. (7). Thus, μ_m and μ_c are the nonlocal multipliers of the metal and ceramic phases, respectively; λ_m and λ_c are the strain gradient multipliers of the metal and ceramic parts, respectively. Accordingly, μ_c/μ_m and λ_c/λ_m, respectively, denote the nonlocal ratio and strain gradient ratio. Further discussion of this issue can be found in the following paragraphs. First, a comparison was made with the work of [19] to validate the vibration frequency of an FG nanobeam based on an NSGT. Therefore, the validation of the first dimensionless vibration frequency $\Omega = \omega L^2\sqrt{\rho_c/E_c}/h$, at different values of nonlocal and strain gradient multipliers, and an excellent agreement can be obtained between the obtained results and [19].

Fig. 2 shows the time history of normalized dynamic deflection at different values of load velocity factor V^* of 0.1, 0.12, and 0.15. Also, in Fig. 2 the nonlocal parameter ratio (μ_c/μ_m) of 0.5, 1, 1.5, and 2 at the FG index $p = 1$ is shown. It is assumed in this research that the ratio of strain gradient multipliers of the metal and ceramic (λ_c/λ_m) is equal 2. For every value of the load velocity factor, an increment in the nonlocal ratio results in higher values of normalized dynamic deflection. This is owing to the reduced stiffness of FG nanobeam when the nonlocal ratio becomes higher. Such behavior indicates that an FG nanobeam displays stiffness-softening effects when the nonlocal ratio increases. Therefore, the dynamic behavior of an FG nanobeam depends on the completion between the nonlocal values of metal and ceramic phases.

Dynamic deflection of the nanosized beam against the load velocity factor based on NETs, Classic Elasticity Theory (CET), and NSGT is plotted in Fig. 3 at $p = 1$ and $t^* = 0.5$. In the case of CET, it is assumed that $\mu_m = \lambda_m = 0$. Moreover, it is considered for the case of NET that $\mu_m = 0.2$ and $\lambda_m = 0$. In this figure, the nonlocal ratio is assumed to be $\mu_c/\mu_m = 2$. The dynamic deflection is significantly affected by the moving load velocity. The dynamic deflection is augmented with the load velocity factor until it overtakes a summit valence and then drops suddenly after this valence (critical velocity). However, the dynamic deflection and summit point are dependent on the values of the nonlocal and strain gradient multipliers. NET provides the maximum value of dynamic deflection owing to the inclusion of nonlocal effects. However, by incorporating the strain gradient effect, the NSGT yielded smaller deflections than the NET.

A comparison between the dynamic deflection (time history) of the FG nanobeam obtained by the power law and Mori-Tanaka models is shown in Fig. 4, assuming that $p = 1$. In this figure, the load Velocity Factor (V^*) is set to 0.12. It can be observed that the Mori-Tanaka model of the

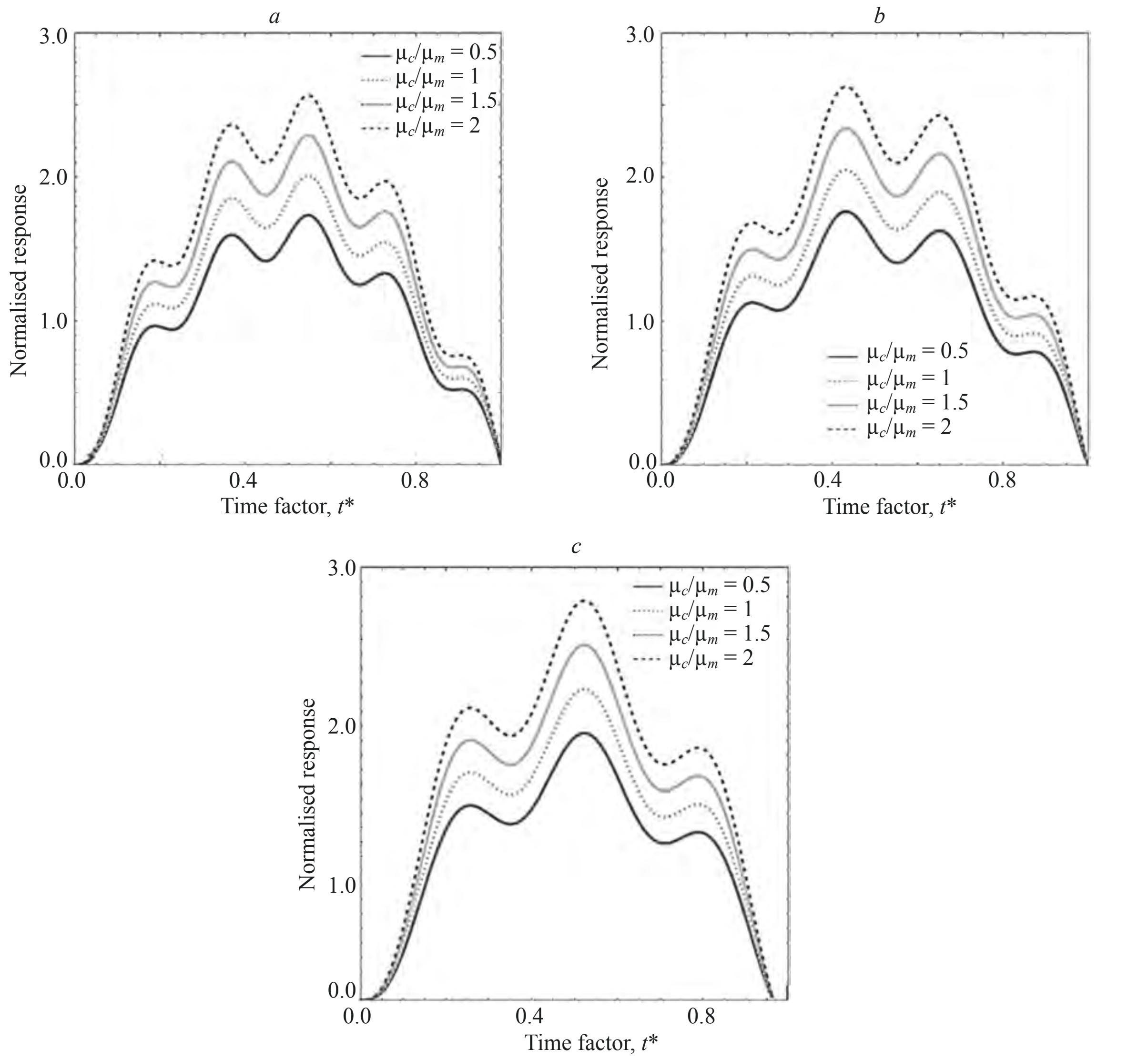

Fig. 2. Time history of normalized dynamic deflection ($\bar{W}$) for varying nonlocal parameter ratios ($\mu_c/\mu_m = 0.5, 1, 1.5, 2$) at distinct load velocity factors: $V^* = 0.1$ (*a*), $V^* = 0.12$ (*b*), $V^* = 0.15$ (*c*). The FG index is $p = 1$, and the strain gradient ratio is fixed at $\lambda_c/\lambda_m = 2$

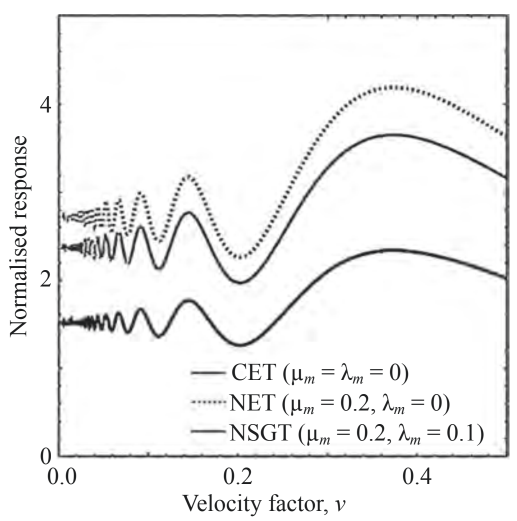

Fig. 3. Nano-thickness unto velocity factor based on distinct elasticity modeling

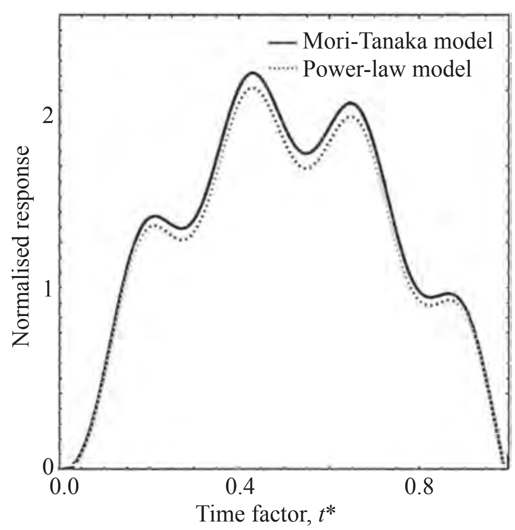

Fig. 4. Thickness beam vs. time factor on distinct gradation modeling

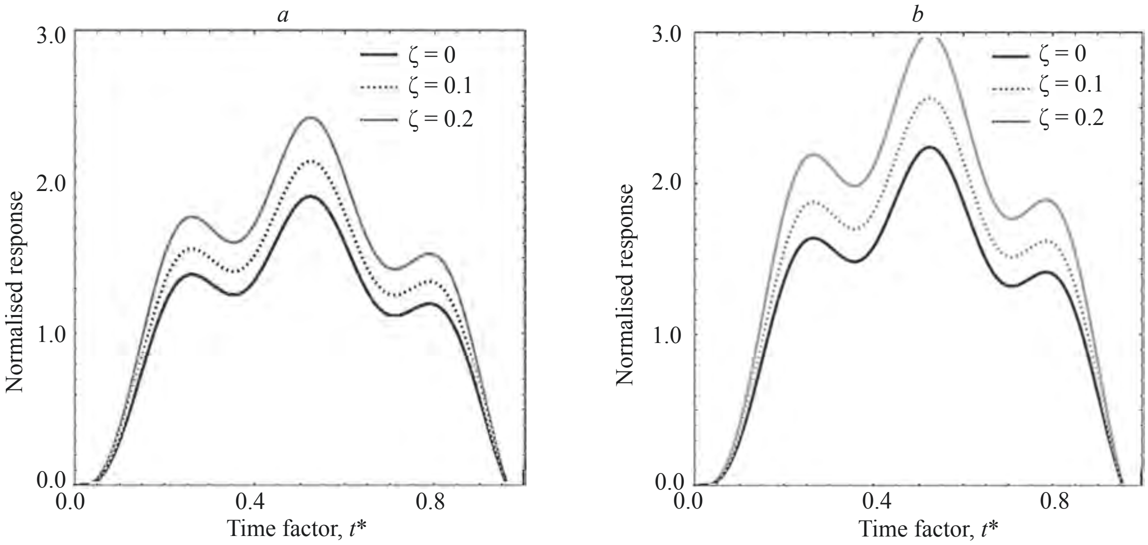

Fig. 5. The nano-thickness beam onto Time factor t^* for p = 0.5 (a), 1 (b)

FG materials results in higher values of dynamic deflection than the power-law model. In particular, the FG nanobeam is more flexible because it describes the material properties using the Mori-Tanaka model which is a more reliable scheme for FG materials than the power-law model.

In Fig. 5, the time history of FG nanobeam has been plotted based on various porosity volume fractions (ζ = 0, 0.1, and 0.2) and FG gradient index (p = 0.5, 1, and 2) at V^* = 0.15. An even porosity dispersion was considered in this figure. The FG nanobeam becomes more flexible at a higher gradient index value because of the higher percentage of the metal constituent compared with the ceramic constituent. Accordingly, the dynamic deflections of the nanobeams increase with increasing gradient index. However, another essential factor in the dynamic response of FG nanobeams is porosity. As the porosity volume (ζ) increases, the value of dynamic deflection becomes higher because porosities inside FG material reduce the structural stiffness.

Conclusions

This article focuses on the dynamic response inspection of a porous Functionally Graded (FG) nanobeam subjected to movable-point loading, considering the effects of graded nonlocality. An altered power-law model was used to investigate the dynamic characteristics of the FG nanobeams, including porosity effects. The nanobeam formulation was based on the higher-order refined beam theory, whereas the side effects were captured according to the Nonlocality Strain Gradient Theory (NSGT). The governing equations were solved using the differential quadrature method and the inverse Laplace transform method. The main findings are summarized as follows:

— An increase in the nonlocal ratio led to higher normalized dynamic deflection values.
— It was reported that NSGT provides smaller deflections than Nonlocal Elasticity Theory.
— The dynamic deflection was augmented with the load velocity until it reached a maximum value, and then dropped after this point, which was called the critical velocity.
— The porosity volume and dynamic deflection increased because the porosity inside the FG material reduced the structural stiffness.

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Authors

Ridha A. Ahmed — Magister, Associate Professor, Associate Professor, Engineering College of Al-Mustansiriyah University, Bahdad, 10001, Iraq, sc 57203618225, https://orcid.org/0000-0002-6666-6564, ridhalwan@uomustansiriyah.edu.iq

Wael N. Abdullah — Magister, Lecturer, Engineering College of Al-Mustansiriyah University, Bahdad, 10001, Iraq, sc 58185482000, https://orcid.org/0000-0001-9599-4279, wael.najm65@gmail.com

Nadhim M. Faleh — D.Sc., Full Professor, Engineering College of Al-Mustansiriyah University, Bahdad, 10001, Iraq, sc 57201261694, https://orcid.org/0000-0003-0003-1643, dr.nadhim@uomustansiriyah.edu.iq

Mamoon A. Al-Jaafari — PhD, Lecturer, Engineering College of Al-Mustansiriyah University, Bahdad, 10001, Iraq, sc 57218543226, https://orcid.org/0009-0005-9597-9825, dr.mamoonal-jaafare@uomustansiriyah.edu.iq

Авторы

Ахмед Ридха А. — магистр, доцент, доцент, Университет Аль-Мустансирия, Багдад, 10001, Ирак, sc 57203618225, https://orcid.org/0000-0002-6666-6564, ridhalwan@uomustansiriyah.edu.iq

Абдулла Ваэль Н. — магистр, преподаватель, Университет Аль-Мустансирия, Багдад, 10001, Ирак, sc 58185482000, https://orcid.org/0000-0001-9599-4279, wael.najm65@gmail.com

Фалех Надим М. — Dr.Sc., профессор, профессор, Университет Аль-Мустансирия, Багдад, 10001, Ирак, sc 57201261694, https://orcid.org/0000-0003-0003-1643, dr.nadhim@uomustansiriyah.edu.iq

Аль-Джафари Мамун Ali — PhD, преподаватель, Университет Аль-Мустансирия, Багдад, 10001, Ирак, sc 57218543226, https://orcid.org/0009-0005-9597-9825, dr.mamoonal-jaafare@uomustansiriyah.edu.iq

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