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Efficiency analysis of planetary gears for electromechanical systems with distributed parameters

Michail E. Lustenkov[1], Andrei P. Korneev[2]✉, Gennady S. Lenevsky[3]

[1,2,3] Belarusian-Russian University, Mogilev, 212030, Belarus

[1] lustenkov@yandex.by, https://orcid.org/0000-0002-4912-3824
[2] ankorn79@mail.ru✉, https://orcid.org/0000-0003-0998-677X
[3] emos@yandex.ru, https://orcid.org/0009-0008-7266-6449

Abstract
Theoretical determination of planetary gear efficiency at design stage for given gear ratio allows for comparison of various designs and selection of most efficient one, providing required kinematic characteristics and minimal power losses. This article examines theoretical determination of planetary gear efficiency for electromechanical systems with distributed parameters for given kinematic design for installed driving and driven links as well as known engagement efficiency of reversing mechanism. Comprehensive analysis of algorithms developed for theoretically estimating planetary gear efficiency is presented. These systems are considered within framework of specific kinematic designs where both driving and driven links are clearly defined, and known engagement efficiency of reversing mechanism is taken into account. Main objective is to provide tools for more accurate and universal determination of planetary gear efficiency. Algorithms for theoretical determination of planetary gear efficiency are developed. Two universal relationships are proposed that allow for estimating energy efficiency of various planetary gear kinematic designs, taking into account functions of driving and driven links. Method enables theoretical evaluation of efficiency at design stage. It is shown that in planetary gearboxes with leading carrier, when selecting kinematic design and gear tooth count to ensure given gear ratio, it is necessary to strive for positive gear ratio. This, all other things being equal, ensures higher efficiency and lower load on gearbox housing. For cycloidal-pinion gearboxes with two-crown satellite with cycloidal tooth profile, central gear, secured in housing, must have greater number of pinions than central gear mounted on driven shaft. Method is applicable to planetary gearboxes and cycloidal-pinion gearboxes operating in reduction and multiplier modes. Obtained results contribute to optimization of gearbox designs and contribute to increased energy savings and improved mechanical efficiency in various applications.

Keywords
planetary gear, reduction gear, multiplier, efficiency, carrier, satellite, gear ratio, electromechanical system, distributed parameters

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Анализ КПД планетарных передач для электромеханических систем с распределенными параметрами

Михаил Евгеньевич Лустенков[1], Андрей Петрович Корнеев[2]✉, Геннадий Сергеевич Леневский[3]

[1,2,3] Белорусско-Российский университет, Могилев, 212030, Беларусь

[1] lustenkov@yandex.by, https://orcid.org/0000-0002-4912-3824
[2] ankorn79@mail.ru✉, https://orcid.org/0000-0003-0998-677X
[3] emos@yandex.ru, https://orcid.org/0009-0008-7266-6449

Аннотация

Введение. Теоретическое определение коэффициента полезного действия (КПД) планетарных передач на стадии проектирования при заданном передаточном отношении позволяет сравнить различные схемы передач и выбрать наиболее рациональную, обеспечивающую необходимые кинематические характеристики и минимальные потери мощности. В работе рассматриваются вопросы теоретического определения КПД планетарных передач для электромеханических систем с распределенными параметрами при известной кинематической схеме, для установленных ведущих и ведомых звеньев, а также известным КПД зацепления реверсивного механизма. Представлен всесторонний анализ алгоритмов, разработанных для теоретической оценки КПД планетарных передач. **Метод.** Представлены инструменты для более точного и универсального определения КПД планетарных передач. Разработаны алгоритмы для теоретического определения КПД планетарных передач на стадии проектирования. Предложены две универсальные зависимости, которые позволяют оценить энергоэффективность различных кинематических схем планетарных передач, учитывая функции ведущего и ведомого звеньев. **Основные результаты.** Показано, что в планетарных передачах с ведущим водилом при выборе кинематической схемы и чисел зубьев колес для обеспечения заданного передаточного отношения необходимо стремиться к его положительному значению, что при равных условиях обеспечивает более высокий КПД и меньшую нагрузку на корпус редуктора. Для циклоидально-цевочных передач с двухвенцовым сателлитом с циклоидальным профилем зубьев центральное колесо, закрепляемое в корпусе, должно иметь большее число цевок, чем центральное колесо, устанавливаемое на ведомом валу. Метод применим для планетарных зубчатых и циклоидально-цевочных передач, работающих в редукторном и мультипликаторном режимах. **Обсуждение.** Полученные результаты вносят вклад в оптимизацию конструкции зубчатых передач, способствуют повышению энергосбережения и улучшению механического КПД в различных областях применения.

Ключевые слова

планетарная передача, редуктор, множитель, КПД, носитель, сателлит, передаточное число, электромеханическая система, распределенные параметры

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Introduction

Increase in equipment productivity a priori leads to increase in operating speeds and decrease in transient processes time in Electromechanical Systems (EMS) with Distributed Parameters (DP). Almost all EMS with DP include mechanical subsystem which is coupled with electric motor using gearbox [1, 2].

In this case, reducing elements efficiency, in particular planetary gears, is of particular interest when improving and/or developing new EMS with DP. This is due to fact that planetary gears in processes of energy exchange between electric motor and mechanical subsystem with DP have properties of dissipative system, which has significant effect on transient processes parameters in EMS with DP [3, 4].

Power losses in planetary gears depend on many parameters, including characteristics of contacting parts materials, acting loads, lubrication system, etc., complex effect of which can be estimated experimentally [5]. At the same time, theoretical determination of planetary gears efficiency at design stage for given gear ratio allows to compare their various schemes and select most rational one, providing necessary kinematic characteristics and minimal power losses [5]. Developed methods for determining efficiency of planetary gears [6–8] suggest estimating power losses based on known efficiency of inverted mechanism; however, these methods have differences in approaches and obtained dependencies.

Thus, in [6] it is stated that efficiency of transmission with single-crown satellite when transmitting rotation from central wheel to carrier and vice versa is the same. According to dependencies given in [7], efficiency of transmission in multiplier mode will be higher than in reduction mode, which contradicts known experimental data. Obtained expressions for efficiency depend on choice of driven link with leading driver [6] and on range of gear ratios [7, 8]. More complex systems are analyzed in [9, 10], where analytical dependencies are proposed for determining efficiency of four-, five-, and six-link planetary mechanisms, but comparative analysis of numerical values for different schemes is not given. Formulas for determining power losses in planetary gears with two degrees of freedom are given in [11, 12]. Studies of efficiency of gear with modified tooth profile are given in [13, 14]. Proposed profile allows one to obtain difference in number of teeth of central wheel and satellite equal to one, which allows one to implement large gear ratios with small dimensions and weight.

Materials and Methods

Planetary gearbox (and multiplier) is mechanism with one degree of freedom, and accordingly has three main links: leading (in), driven (out) and stopped (stop). Efficiency in engagements is estimated by formula:

$$\eta_{in\text{-}out}^{stop} = -\frac{P_{out}}{P_{in}} = -\frac{T_{out}\omega_{out}}{T_{in}\omega_{un}}, \quad (1)$$

where P_{in}, P_{out} are power; T_{in}, T_{out} are torque; ω_{in}, ω_{out} are angular velocity of links.

This formula follows from power balance equation: $P_{in} + P_{out} + P_{stop} = 0$, or $T_{in}\cdot\omega_{in}\cdot\eta_{in\text{-}out}^{stop} + T_{out}\cdot\omega_{out} + T_{stop}\cdot\omega_{stop} = 0$. At $\omega_{stop} = 0$ you will get formula (1).

Formula (1) for determining efficiency expressed not through useful and expended work, but through their derivatives with respect to time — power, is more convenient for calculations, since values of work can grow indefinitely over time due to increase in values of rotation angles. In this case, expression (1) determines instantaneous efficiency, but it is also applicable to calculating average efficiency values at constant transmission shaft speeds. In this case, we do not consider power losses in bearings and for mixing oil (hydraulic losses) due to specifics of their definition.

When determining efficiency, we will accept several conditions. Driving shaft rotates in positive direction (relative to transmission axis, which has direction from driving shaft to driven shaft). Positive torque is applied to driving shaft, i.e., directions of torque and angular velocity vectors coincide. Direction of rotation of driven shaft is established after determining gear ratio. Direction of torque vector on driven shaft is opposite to direction of angular velocity vector of driven shaft (moment of useful resistance). Due to fact that directions of velocity and torque vectors on driving shaft always coincide, and on driven shaft are always opposite, ratios of powers at output and input of mechanism will always be negative. Efficiency, unlike vector speeds and torques, is scalar positive value that varies from zero to one. Minus sign in expression (1) indicates that input power is positive while output power is negative. This eliminates uncertainty and ensures that efficiency is always positive, as zero and negative values of efficiency in wedge mechanisms, determined based on force diagrams considering frictional forces, indicate self-locking behavior.

To determine efficiency of transmission when using different planetary gear schemes, we assume that efficiency is known η_0 reversed mechanism, i.e., transmission with driver stopped. It is equal to product of efficiency n couplings connected in mechanism with stopped carrier in series: $\eta_0 = \eta_1\cdot\ \eta_2\cdot\ldots\cdot\ \eta_n$.

Proposed method additionally uses two dependencies. First is vector condition of moments equilibrium acting on planetary gear links along transmission axis.

$$T_{in} + T_{out} + T_{stop} = 0. \quad (2)$$

Formula (2) is named Willis formula, which determines gear ratio of reverse mechanism with carrier stopped

$$\frac{\omega_{in} - \omega_h}{\omega_{out} - \omega_h} = i_0^h, \quad (3)$$

where ω_h is angular velocity of carrier; i_0^h is gear ratio of reverse mechanism.

Let's consider overview of algorithm for determining efficiency of planetary gear system. Input parameters are kinematic diagram of planetary gear system and the efficiency of inverse mechanism, denoted as η_0.

1. We determine gear ratio of inverse mechanism i_0^h based on number of teeth, assuming that function of driving link is preserved. We take stationary link in studied mechanism as driven link in inverse mechanism. If driving link is carrier, we preserve function of driven link, while stationary link in inverse mechanism will be considered as driving link.
2. From formula (3) express gear ratio for circuit under consideration through i_0^h, substituting obtained value into i_0^h, expressed through number of teeth, we determine directions of angular velocity and torque vectors on driven link.
3. Based on vector equation of moment balance (2), derive scalar expression (projection of vector equation onto an axis), where moments are substituted considering their signs. From this expression, find torque on stationary link in inverse mechanism (torque on driver).
4. Using formula (1), obtain expression for efficiency of inverse mechanism η_0. From it express ratio of moments on leading and driven links through i_0^h and η_0.
5. According to expression (1), compile dependence for transmission efficiency for circuit under consideration, expressed through i_0^h and η_0.

Let us consider methodology using examples of most common types of planetary gears (Fig. 1).

Efficiency of planetary gear with 2*k-h* scheme and single-crown satellite

Fig. 1, shows simple planetary gear with single-crown satellite (satellites), designed according to 2*k-h* scheme with central wheels *a* and *b* and planet carrier *h* (Fig. 1, *a*). Most commonly used scheme is with leading sun wheel *a*, fixed crown wheel *b* and driven planet carrier *h*. Directions of moments and angular velocities of leading and driven links are shown schematically in Fig. 2, *a*. Known efficiencies are η_{ag} external engagement of wheel *a* and satellite *g* and efficiency η_{gb} internal engagement of satellite *g* and central wheel *b* in inverted mechanism. Accordingly, efficiency of inverted mechanism is known: $\eta_0 = \eta_{ag}\cdot\eta_{gb}$.

In reversed mechanism, retain leading wheel *a*, and driven wheel *b* becomes. Gear ratio of reversed mechanism is

$$i_{ab}^h = \frac{\omega_a}{\omega_b} = -\frac{z_g}{z_a}\cdot\frac{z_b}{z_g} = -\frac{z_b}{z_a}. \quad (4)$$

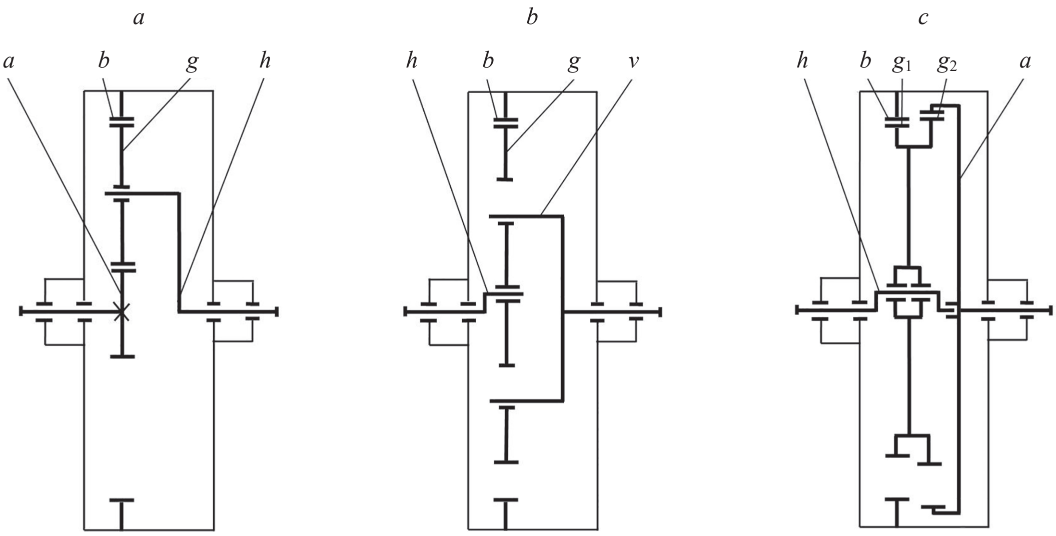

Fig. 1. Planetary gear diagrams:
2*k*-*h* with a single-crown satellite (*a*); *k*-*h*-*v* with single-crown satellite (*b*); 2*k*-*h* with double-crown satellite (*c*)

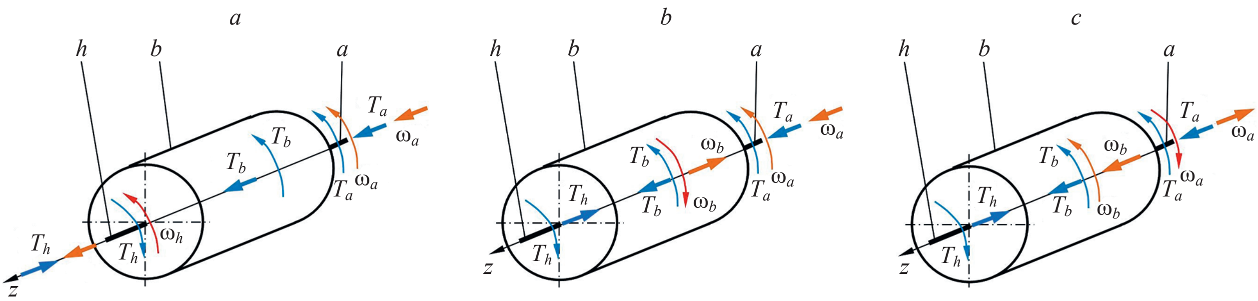

Fig. 2. Directions of angular velocities and moments in 2*k*-*h* transmission with single-crown satellite: when transmitting rotation $a \to h$ (*a*); in inverted mechanism $a \to b$ (*b*); when transmitting rotation $b \to h$ (*c*)

Direction of angular velocities and moments is shown in Fig. 2, *b*. From formula (3), taking into account that $\omega_b = 0$, receive

$$\frac{\omega_a - \omega_h}{-\omega_h} = i_{ab}^h,\ -\frac{\omega_a}{\omega_h} + 1 = i_{ab}^h,\ i_{ab}^h = 1 - i_{ah}^b. \qquad (5)$$

Substituting expression (4) into formula (5), obtain:

$$i_{ah}^b = 1 - i_{ab}^h = 1 + \frac{z_b}{z_a}. \qquad (6)$$

From formula (6) it is clear that $i_{ah}^b > 1$ for any ratio of teeth number z_a and z_b. Consequently, direction of angular velocity vector of driven link coincides with direction of angular velocity vector of driving link, since $\omega_h = \omega_a / i_{ah}^b$. Moment of useful resistance T_h will, accordingly, have opposite sign (Fig. 2, *a*).

Based on equation of moments equilibrium, determine direction and magnitude of moment on stopped link. In accordance with formula (4), gear ratio of reversed mechanism $i_{ab}^h \le -1$. Limit value is equal to –1, achieved when number of teeth is equal $z_a = z_b$. This is possible in transmissions with bevel gears, which have angle $\Theta = \pi/2$ rad (Fig. 3, *c*). At $z_a > z_b$ we obtain transmission scheme similar to those shown in Fig. 1, *a*, Fig. 3, *a*, or Fig. 3, *b*, in which larger wheel is driving wheel. This case is further considered separately.

From formula (5) follows that gear ratio $i_{ah}^b \ge 2$, negative moment on driven link will be greater than moment on leading link in absolute value, therefore equality of sum of moments on axle to zero will also be ensured by positive value of moment on stopped link. In general

$$T_a + T_b + T_h = 0,\ T_h = -(T_a + T_b). \qquad (7)$$

Expression for efficiency of inverted mechanism according to formula (1)

$$\eta_0 = -\frac{P_b}{P_a} = -\frac{T_b\omega_b}{T_a\omega_a} = -\frac{T_b}{T_a i_{ab}^h}, \quad -\eta_0 i_{ab}^h = \frac{T_b}{T_a}. \qquad (8)$$

According to formula (1), with substitution of expressions (7), (8) and (5) into it, efficiency of transmission designed according to circuit under consideration is equal to

$$\eta_{ah}^b = -\frac{P_h}{P_a} = -\frac{T_h\omega_h}{T_a\omega_a} = -\frac{-(T_a + T_b)}{T_a}\cdot\frac{1}{i_{ah}^b} = (1 - \eta_0 i_{ab}^h)\frac{1}{i_{ah}^b} = \frac{1 - \eta_0 i_{ab}^h}{1 - i_{ab}^h}. \qquad (9)$$

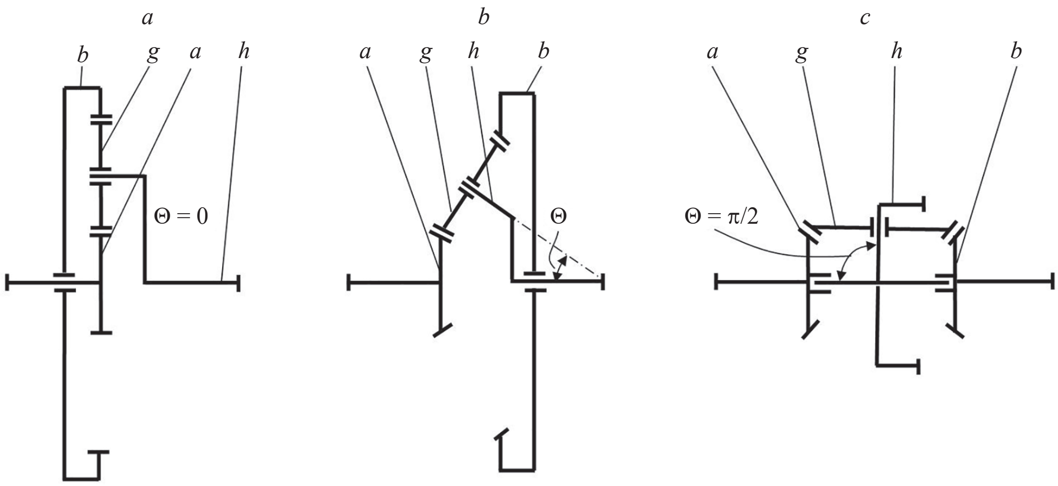

Fig. 3. Diagrams of planetary transmission with cylindrical (*a*) and bevel (*b*, *c*) wheels

As example, let's consider three-satellite planetary gear with parameters $z_a = 21$, $z_b = 63$. Efficiency of internal gearing is higher than that of external gearing, so we will accept: $\eta_{ag} = 0.97$, $\eta_{gb} = 0.98$. At the same time, $\eta_0 = \eta_{ag} \cdot \eta_{gb} = 0.951$.

Gear ratios of reversed and considered gears $i^h_{ab} = -3$, $i^b_{ah} = 4$. Transmission efficiency $\eta^b_{ah} = 0.963$ and it is higher than efficiency of reverse transmission.

With driving wheel b and driven driver h, directions of angular velocities and moments for reversed mechanism (b is driving, a is driven, h is stopped link) are shown in Fig. 2, *c*. Transmission efficiency is determined by formula similar to dependence (9)

$$\eta^a_{bh} = \frac{1 - \eta_0 i^h_{ba}}{1 - i^h_{ba}}. \tag{10}$$

Gear ratios, while

$$i^h_{ba} = -\frac{z_a}{z_b}; \quad i^a_{bh} = 1 - i^h_{ba} = 1 + \frac{z_a}{z_b}.$$

Efficiency of mechanism with above parameters is equal to $\eta^a_{bh} = 0.988$.

With leading central wheel with large number of teeth and driven carrier, efficiency can be calculated using both formula (9) and formula (10), designating larger wheel as and smaller wheel as b. In general case, dependencies (9) and (10) can be written as

$$\eta^{stop}_{in\text{-}h} = \frac{1 - \eta_0 i^h_{in\text{-}stop}}{1 - i^h_{in\text{-}stop}} = \frac{i^h_{stop\text{-}in} - \eta_0}{i^h_{stop\text{-}in} - 1}. \tag{11}$$

At same time

$$i^h_{in\text{-}stop} = -\frac{z_{stop}}{z_{in}}; \quad i^h_{stop\text{-}in} = \frac{1}{i^h_{in\text{-}stop}} = -\frac{z_{in}}{z_{stop}}, \tag{12}$$

where z_{in}, z_{stop} are number of driving teeth and fixed wheels, respectively.

With leading driver and driven wheel a, obtain a multiplier transmission. When determining gear ratio of reversed mechanism, function of driven link is retained for driven link of transmission in question (wheel a), leading link is wheel b — stopped in circuit in question: $i^h_{ba} = \omega_b/\omega_a = -z_a/z_b$.

From Willis formula (equation (3)) gear ratio is equal to $i^b_{ha} = i^h_{ba}/(i^h_{ba} - 1)$. Expressed in terms of number of teeth, it will take form $i^b_{ha} = z_a/(z_a + z_b)$. Equation (8) is transformed to the form: $T_a/T_b = -i^h_{ba} \cdot \eta_0$.

Transmission efficiency is

$$\eta^b_{ha} = -\frac{P_a}{P_h} = -\frac{T_a \omega_a}{-(T_a + T_b)\omega_h} = \frac{1}{\left(1 - \frac{1}{i^h_{ba}\eta_0}\right) i^b_{ha}} = \frac{i^h_{ba} - 1}{i^h_{ba} - \frac{1}{\eta_0}}. \tag{13}$$

Gear ratios of reversed mechanisms with different driving links (wheel a or b) are related to each other as inverse quantities: $i^h_{ab} = 1/i^h_{ba}$. Let us express gear ratio of transmission under consideration through i^h_{ab} to compare efficiency of mechanism with leading carrier and driven carrier using expression (9).

$$\eta^b_{ha} = \frac{1 - i^h_{ab}}{1 - \frac{i^h_{ab}}{\eta_0}}.$$

In general, we write

$$\eta^{stop}_{h\text{-}out} = \frac{1 - i^h_{out\text{-}stop}}{1 - \frac{i^h_{out\text{-}stop}}{\eta_0}} = \frac{i^h_{stop\text{-}out} - 1}{i^h_{stop\text{-}out} - \frac{1}{\eta_0}}, \tag{14}$$

where gear ratios are determined from expression

$$i^h_{out\text{-}stop} = -\frac{z_{stop}}{z_{out}}, \quad i^h_{stop\text{-}out} = -\frac{z_{out}}{z_{stop}}. \tag{15}$$

Comparing expressions (11) and (14), note that they are not identical. Similar conclusion can be made based on dependencies given in [3]. Let us analyze obtained expressions (11) and (14) in range of gear ratios from –1 to –16 (practically used range).

As can be seen from graphs, with high efficiency in engagements ($\eta_{ag} = 0.97$, $\eta_{gb} = 0.98$, $\eta_0 = 0.95$) efficiency dependencies for gear and multiplier modes are practically same, difference is observed only in thousandths. With low efficiency in engagements ($\eta_{ag} = 0.86$, $\eta_{gb} = 0.87$, $\eta_0 = 0.75$) Transmission efficiency also decreases with increase in absolute values of gear ratios, but decrease in efficiency in multiplier mode occurs more sharply.

Efficiency of planetary transmission according to *k-h-v* scheme with single-crown satellite

Kinematic diagram of transmission is shown in Fig. 1, *b*.

In reduction mode, planet carrier h is the leading one, and the satellite g is driven one. Transmission efficiency will be determined by product of efficiency in engagement and efficiency of mechanism v for removing rotation from satellite to driven shaft. We consider only efficiency of engagement, since losses in mechanism depend on its design (Hooke's clutch, parallel crank mechanism, etc.). We consider reversed mechanism with stopped planet carrier with preserved function of driven link g and driving wheel b. Its gear ratio is

$$i_{bg}^h = \frac{\omega_b}{\omega_g} = \frac{z_g}{z_b}. \qquad (16)$$

Dependence of planetary gears efficiency with single-crown satellites (2*k*-*h* diagram) on gear ratios in reverse gear is shown in Fig. 4.

From formula (3), taking into account that $\omega_b = 0$, $i_{ha}^b = 1/i_{ah}^b$ and $i_{gb}^h = 1/i_{bg}^h$, we get

$$\frac{-\omega_h}{\omega_g - \omega_h} = i_{bg}^h; \quad i_{hg}^b = \frac{\omega_h}{\omega_g} = \frac{i_{bg}^h}{i_{bg}^h - 1} = \frac{1}{1 - i_{gb}^h} = \frac{z_g}{z_g - z_b}. \qquad (17)$$

Because $z_b > z_g$ (in cylindrical transmissions) gear ratio of transmission in question will be negative, angular velocities of carrier and satellite will be in different directions, moments on carrier and satellite will coincide. Efficiency of inverted mechanism

$$\eta_0 = -\frac{T_g\omega_g}{T_b\omega_b} = -\frac{T_g}{T_b i_{bg}^h}, \quad \frac{T_b}{T_g} = -\frac{1}{i_{bg}^h \eta_0}. \qquad (18)$$

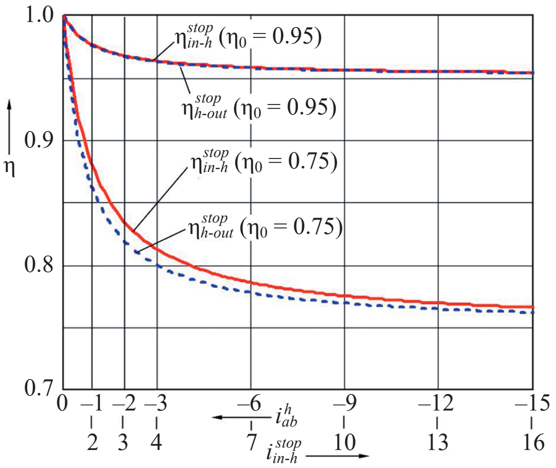

Fig. 4. Dependence of planetary gears efficiency with single-crown satellites (2*k*-*h* diagram) on gear ratios in reverse gear

Taking into account substitution of transformed expression (7): $T_h = -(T_g + T_b)$, and dependencies (17) and (18), efficiency of gear transmission is determined

$$\eta_{hg}^b = -\frac{P_g}{P_h} = -\frac{T_g\omega_g}{-(T_g + T_b)\omega_h} = \frac{i_{bg}^h - 1}{i_{bg}^h - \frac{1}{\eta_0}} = \frac{1 - i_{gb}^h}{1 - \frac{i_{gb}^h}{\eta_0}}. \qquad (19)$$

Reasoning similarly, it can be shown that for transmission with leading carrier and stopped link g, formula for efficiency is η_{hb}^g will be written as expression (19) with modified subscripts at gear ratios: "*gb*" is replaced by "*bg*", and vice versa. In general, expression for determining efficiency $\eta_{h\text{-}out}^{stop}$ for *k*-*h*-*v* transmission will be described by dependence (14), as well as for 2*k*-*h* transmission. Gear ratios of inverted mechanisms for *k*-*h*-*v* scheme will be positive, in contrast to those calculated by formula (15).

$$i_{stop\text{-}out}^h = \frac{1}{i_{out\text{-}stop}^h} = \frac{z_{out}}{z_{stop}}.$$

When operating in multiplier mode, carrier becomes driven link. Having carried out calculations according to considered algorithm for two variants with leading links that retain their functions in reversed mechanism, we obtain

$$i_{gh}^b = 1 - i_{gb}^h, \quad i_{bh}^g = 1 - i_{bg}^h.$$

$$\frac{T_b}{T_g} = -i_{gb}^h \eta_0; \quad \frac{T_g}{T_b} = -i_{bg}^h \eta_0.$$

For transmission variant according to *k*-*h*-*v* scheme with leading satellite, we get

$$\eta_{g\text{-}h}^b = \frac{1 - i_{gb}^h \eta_0}{1 - i_{gb}^h}.$$

In general, expression for determining efficiency of transmission according to *k*-*h*-*v* scheme will be described by dependence (11), and gear ratio of inverted mechanism will differ from formula (12) by absence of a minus

$$i_{in\text{-}stop}^h = \frac{z_{stop}}{z_{in}}.$$

Let us conduct numerical analysis of practically significant case — cycloidal pinion transmission with difference in numbers of central wheel pinions and teeth of satellite equal to one with following parameters: $z_b = 16$, $z_g = 15$ and $\eta_0 = 0.98$. Efficiency in gear mode was $\eta_{ha}^b = 0.75$, Efficiency in multiplier mode $\eta_{ha}^b = 0.68$. Gear ratio (multiplication factor) is –15. Let's compare it with efficiency of 2*k*-*h* transmission with single-crown satellite with same gear ratio module and efficiency of reversed mechanism. In this case, in reduction and multiplier modes, in accordance with formulas (11) and (14), efficiency of transmission is 0.98. Despite fact that in 2*k*-*h* transmission there are two engagements and losses in reversed mechanism will be higher ($\eta_0 = 0.95$), efficiency of transmission will also decrease slightly and will be equal to 0.95. At same time, efficiency of transmission according to *k-h-v* scheme will also be lower, since losses in mechanism of motion removal to driven shaft are not taken into account. Thus, transmission according to *k-h-v* scheme is significantly inferior to transmission according to 2*k-h* scheme with single-crown satellite in terms of energy efficiency. It should be additionally noted that considered transmissions according to 2*k-h* scheme with single-crown satellites are not used at gear ratios above 12.5–15.

Let us analyze dependence of efficiency of transmission according to *k-h-v* scheme on gear ratio of reversed mechanism and losses in its engagement (Fig. 5). Gear ratio of reversed mechanism is selected as variable parameter $i_{bg}^{h} = z_g/z_b$, theoretical range of change of which is 0–1. Graph has been supplemented with dependencies of absolute values of gear ratios of gears according to *k-h-v* scheme in practically applicable range up to 80.

From graphical dependencies it follows that transmission according to *k-h-v* scheme in multiplier mode can be used under condition $i_{bg}^{h} < \eta_0$. At efficiency values equal to or greater than gear ratio of reversed mechanism, when operating in multiplier mode, phenomenon of self-braking occurs. Relatively high values of multipliers efficiency (but not exceeding η_0) possible at small values i_{bg}^{h}, which is ensured by small diameter of satellite in relation to diameter of central wheel. With such ratios, it is difficult to design mechanism for transmitting motion from drive shaft to satellite (mechanism *v*). Fig. 6 shows change in efficiency of transmission according to *k-h-v* scheme from the absolute value of gear ratio in reduction and multiplier modes with different values of power losses in reversed mechanism.

It is also possible to estimate efficiency of cycloidal-pinion gear which is special case of gears according to *k-h-v* scheme. In epicycloidal gear, difference in numbers of pinions and teeth is equal to one: $z_b - z_g = 1$. According to formula $z_g = -i_{hg}^{b}$. Thus, in Fig. 6, values of pinion wheel (satellite) teeth number will be plotted along abscissa axis.

Efficiency of transmission 2*k-h* with two-crown satellite

Use of these diagrams (Fig. 1, *c*) allows to significantly expand range of gear ratios. Gear ratio with leading driver is determined as

$$i_{ha}^{b} = \frac{\omega_h}{\omega_a} = \frac{i_{ba}^{h}}{i_{ba}^{h} - 1} = \frac{1}{1 - i_{ab}^{h}}.$$

At same time

$$i_{ba}^{h} = \frac{1}{i_{ab}^{h}} = \frac{z_a z_{g1}}{z_b z_{g2}},$$

for cycloidal pinion gears $z_{g1} = z_b - 1$, $z_{g2} = z_a - 1$. That's why

$$i_{ha}^{b} = \frac{z_a(z_b - 1)}{z_b - z_a}. \tag{20}$$

Efficiency with leading driver is estimated by formula (14). However, it needs to be adjusted, since in case of 2*k-h* transmission and *k-h-v* transmission with single-crown satellite (formula (19)), in reverse motion, central wheel retained function of driven link. In transmission with double-crown satellite, with different combinations of teeth numbers, several options are possible. With leading driver, efficiency of transmission, in accordance with formula (13), is written

$$\eta_{ha}^{b} = -\frac{P_a}{P_h} = -\frac{T_a\omega_a}{-(T_a + T_b)\omega_h} = \frac{1}{\left(1 + \dfrac{T_b}{T_a}\right) i_{ha}^{b}}. \tag{21}$$

Let us consider two possible cases of transmission power loading. In the first case (Fig. 7, *a*), transmission ratio $i_{ha}^{b} < 0$. Direction of driving rotation and driven shafts is opposite. Direction of moments will coincide. In the second case, gear ratio i_{ha}^{b} positively (Fig. 7, *b*), directions of driving rotation and driven shafts coincide, therefore directions of moments acting on them are opposite.

Ratio of moments T_b/T_a in expression (21) is determined from expression for efficiency of inverted mechanism. In this case, links of mechanism are given speed equal to $-\omega_h$. In first case, angular velocity ω_a will retain (and increase) its negative value, in which, given positive value of moment, T_a corresponds to function of the driven link (different directions of torque and angular velocity). Efficiency of reversed mechanism will be determined by dependence $\eta_0 = P_a/P_b$. In the second case

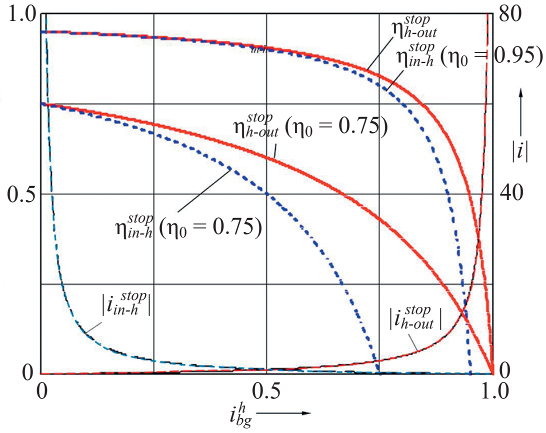

Fig. 5. Dependence of efficiency of *k-h-v* gears on gear ratios

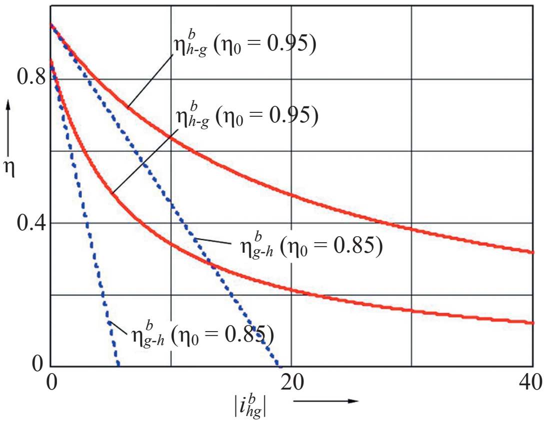

Fig. 6. Dependence of gears according efficiency to *k-h-v* scheme on gear ratios and losses in reverse mechanism

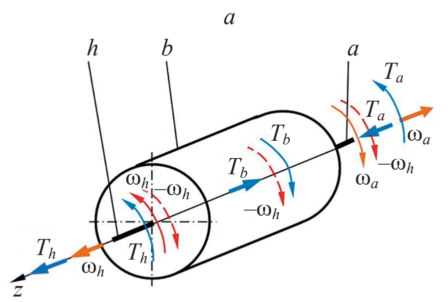

Fig. 7. Directions of angular velocities and moments in $2k$-h transmission with two-crown satellite: with driven link a in reversed mechanism (a); with leading link a in reversed mechanism (b)

(Fig. 7, b), $\omega_a - \omega_h > 0$ link a will also retain slave function if this condition is violated, i.e., if $\omega_a - \omega_h < 0$, direction of rotation will change and coincide with direction of moment. Link a will become leading one and efficiency will be determined by dependence $\eta_0 = P_b/P_a$.

After carrying out transformations similar to those considered above, we obtain for first and second cases, respectively

$$\frac{T_b}{T_a} = -\frac{1}{\eta_0 i_{ba}^h}; \quad \frac{T_b}{T_a} = -\frac{\eta_0}{i_{ba}^h}.$$

In the first case, formula (13) with similar transformations will be valid for estimating efficiency, and in general, formula (14) will be valid. In the second case, condition $\omega_a - \omega_h < 0$ can be reduced to form $i_{ha}^b > 1$. Then transmission efficiency can be estimated using formula (11), swapping numerator and denominator and replacing link indices "in" with "out". In general, formula (14) will be transformed to form

$$\eta_{h\text{-}out}^{stop} = \frac{1 - i_{out\text{-}stop}^h}{1 - \dfrac{i_{out\text{-}stop}^h}{\eta_0^{\mathrm{sign}(1 - i_{h\text{-}out}^{stop})}}} = \frac{i_{stop\text{-}out}^h - 1}{i_{stop\text{-}out}^h - \dfrac{1}{\eta_0^{\mathrm{sign}(1 - i_{h\text{-}out}^{stop})}}}. \quad (22)$$

Second case corresponds to multiplier mechanism with two-link satellite and leading carrier, implementation of which in transmissions according to $2k$-h scheme with single-ring satellites is impossible. If necessary, it is possible to express efficiency of transmissions only through gear ratios of generalized mechanisms, making appropriate replacements of stop-out gear ratio in expression (22) in accordance with formulas (17):

$$i_{h\text{-}out}^{stop} = \frac{1}{1 - i_{out\text{-}stop}^h} = \frac{i_{stop\text{-}out}^h}{i_{stop\text{-}out}^h - 1}. \quad (23)$$

Results and discussion

Let us analyze expression (22), expressing efficiency of transmission with leading carrier through efficiency of inverted mechanism and gear ratio is stop-out in range of its variations –100...100. Efficiency graph will have two branches, decreasing with increase in absolute values of abscissa, with maximum at $i_{h\text{-}out}^{stop} = 1$. At this value, efficiency is equal to one, which indicates that there is no reduction or multiplication of speed, driving and driven shafts do not perform relative movements, rotating as single body. For clarity and ability to compare results, we will reflect left branch of graph, where $i_{h\text{-}out}^{stop} < 1$ on its right side relative to point with maximum value $i_{h\text{-}out}^{stop} = 1$, while plotting ordinate values obtained using formula (23) with abscissas corresponding to values $-i_{h\text{-}out}^{stop} + 2$ (Fig. 8).

Thus, absolute values of gear ratios are plotted along abscissa axis. In Fig. 8, efficiency values are shown by solid lines at $i_{h\text{-}out}^{stop} > 1$, dashed at $i_{h\text{-}out}^{stop} < 1$.

Comparative analysis shows that it is advisable to implement same value of gear ratio (absolute value) with unidirectional rotation of driving and driven shafts, which for cycloidal-pinion gears, based on formula (20), leads to condition $z_b > z_a$. Efficiency will be higher and difference will be greater lower efficiency of reversed mechanism. An additional advantage will be lower moment on housing (stopped link), since according to diagram in Fig. 7, moment on stationary wheel compensates for combined action of two unidirectional moments T_a and T_h: $T_b = T_h - T_a$. In case of oppositely directed rotation of driving and driven shafts, moment on housing is greater, with same kinematic characteristics: $T_b = T_h + T_a$.

Theoretical possibility of obtaining multiplier mechanisms with high efficiency in range of gear values –1...1 is not realized by some schemes. Analysis of formula (20), for example, shows that specified range cannot be obtained with any ratios z_a and z_b.

Similarly, expression (11) can be generalized for $2k$-h gears with leading central wheel and driven carrier. When

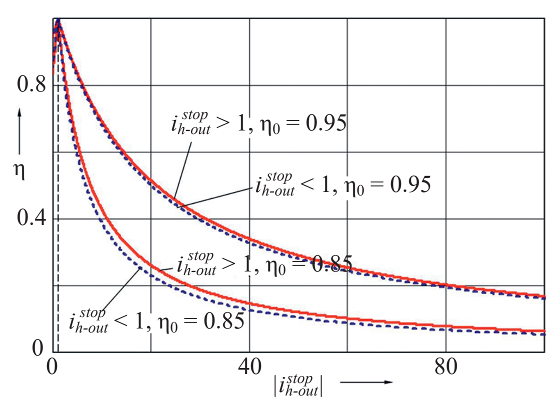

Fig. 8. Dependence of transmission efficiency with two-crown satellite and leading carrier on transmission ratio module and efficiency of inverted mechanism

$0 < i_{in\text{-}h}^{stop} < 1$ the driving wheel in reverse mechanism becomes driven, which must be taken into account. Formula (11) in general form

$$\eta_{in\text{-}h}^{stop} = \frac{1 - \eta_0^{\operatorname{sign}\left(\frac{i_{in\text{-}h}^{stop}-1}{i_{in\text{-}h}^{stop}}\right)} i_{in\text{-}stop}^{h}}{1 - i_{in\text{-}stop}^{h}} =$$

$$= \frac{i_{stop\text{-}in}^{h} - \eta_0^{\operatorname{sign}\left(\frac{i_{in\text{-}h}^{stop}-1}{i_{in\text{-}h}^{stop}}\right)}}{i_{stop\text{-}in}^{h} - 1}. \quad (24)$$

Fig. 9 shows efficiency graphs. Its change, depending on gear ratio, occurs at three intervals: $i_{in\text{-}h}^{stop} < 0$, $0 < i_{in\text{-}h}^{stop} < 1$ and $i_{in\text{-}h}^{stop} > 1$. Change in efficiency at gear ratios in third interval has already been investigated and is shown in Fig. 4.

With increase in absolute values of gear ratios, efficiency graphs asymptotically approach value η_0, at $i_{in\text{-}h}^{stop} < 0$ increasing, at $i_{in\text{-}h}^{stop} > 1$ decreasing.

Obtained results can be used in improvement and/or development of EMS with DP to improve operational characteristics [15, 16].

Overall, these findings underscore importance of meticulous design choices in planetary gear systems. By focusing on optimizing gear ratio and strategically configuring teeth number and pinions, engineers can achieve superior performance metrics, including increased efficiency and reduced mechanical stress on gearbox components. Future research should continue to explore these relationships further, potentially leading to even more refined methodologies for enhancing planetary transmission efficiencies.

Dependencies for planetary gears efficiency theoretical determination designed according to various configurations have been systematically derived. These dependencies are characterized by utilization of only two fundamental formulas (22) and (24), which hinge on two pivotal parameters: gear ratio and the efficiency of reversed mechanism.

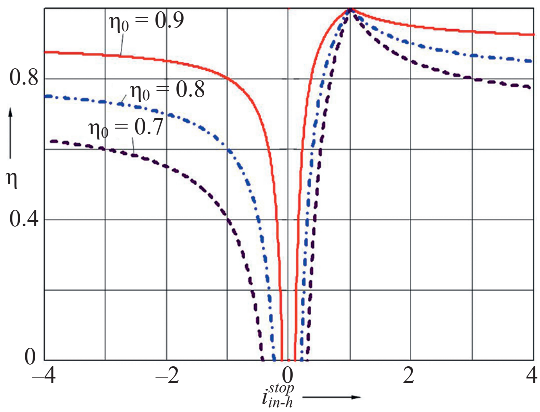

Fig. 9. Dependence of gears efficiency according to 2*k*-*h* scheme with driven carrier on gear ratios and efficiency of inverted mechanism

Conclusion

Through comprehensive comparative analysis, it has been established that in planetary gear systems featuring leading carrier, careful selection of kinematic scheme and teeth number on gears is paramount to achieving desired gear ratio. It is essential to prioritize positive value for this gear ratio; such choice significantly enhances overall efficiency of system. Furthermore, positive gear ratio contributes to reduction in load experienced by gearbox housing, thereby improving durability and reliability of transmission system.

In particular, for cycloidal-pinion gear configurations that incorporate two-crown satellite with cycloidal tooth profile, it is crucial to ensure that central wheel, which is fixed within housing, possesses greater number of pinions compared to central wheel that is mounted on driven shaft. Design consideration not only optimizes load distribution among components but also enhances gear system operational efficiency.

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Authors

Michail E. Lustenkov — D.Sc., Professor, Rector, Belarusian-Russian University, Mogilev, 212030, Belarus, sc 36545142000, https://orcid.org/0000-0002-4912-3824, lustenkov@yandex.by

Andrei P. Korneev — Senior Lecturer, Belarusian-Russian University, Mogilev, 212030, Belarus, sc 57463582100, https://orcid.org/0000-0003-0998-677X, ankorn79@mail.ru

Gennady S. Lenevsky — PhD, Associate Professor, Associate Professor, Belarusian-Russian University, Mogilev, 212030, Belarus, sc 58631794500, https://orcid.org/0009-0008-7266-6449, emos@yandex.ru

Авторы

Лустенков Михаил Евгеньевич — доктор технических наук, профессор, ректор, Белорусско-Российский университет, Могилев, 212030, Беларусь, sc 36545142000, https://orcid.org/0000-0002-4912-3824, lustenkov@yandex.by

Корнеев Андрей Петрович — старший преподаватель, Белорусско-Российский университет, Могилев, 212030, Беларусь, sc 57463582100, https://orcid.org/0000-0003-0998-677X, ankorn79@mail.ru

Леневский Геннадий Сергеевич — кандидат технических наук, доцент, доцент, Белорусско-Российский университет, Могилев, 212030, Беларусь, sc 58631794500, https://orcid.org/0009-0008-7266-6449, emos@yandex.ru

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