НАУЧНО-ТЕХНИЧЕСКИЙ ВЕСТНИК ИНФОРМАЦИОННЫХ ТЕХНОЛОГИЙ, МЕХАНИКИ И ОПТИКИ
сентябрь–октябрь 2026 Том 26 № 5 http://ntv.ifmo.ru/
SCIENTIFIC AND TECHNICAL JOURNAL OF INFORMATION TECHNOLOGIES, MECHANICS AND OPTICS
September–October 2026 Vol. 26 No 5 http://ntv.ifmo.ru/en/
ISSN 2226-1494 (print) ISSN 2500-0373 (online)

doi: 10.17586/2226-1494-2026-26-5-935-944

Robust adaptive compensation of unknown multi-harmonic disturbances in multivariable linear systems

Cong Vinh Tu[1]✉, Natalia A. Dudarenko[2]

[1,2] ITMO University, Saint Petersburg, 197101, Russian Federation

[1] congvinhvkdn@gmail.com✉, https://orcid.org/0009-0005-5106-8560
[2] dudarenko@itmo.ru, https://orcid.org/0000-0002-3553-0584

Abstract
This paper proposes a new direct adaptive disturbance compensation method for multivariable linear systems, which is robust with respect to external disturbances. The external disturbances are assumed to be multi-harmonic signals (regular component) with additional random noise (irregular component), where the bias, phase, amplitude, and frequency are all unknown. The objective is to design a state-feedback control law capable of regulating the system output to zero in the absence of random noise. The robustness of the closed-loop system with respect to additive unmodeled disturbances is reflected in the fact that the proposed control law guarantees boundedness of all closed-loop signals. To solve this problem, first, the system inputs and outputs are decoupled using Falb–Wolovich approach. Then, the direct disturbance compensation method based on the internal model principle is applied to the resulting decoupled system. As a result, the adaptive algorithm can be constructed robustly and independently for each control channel. The adaptive algorithm with memory regressor extension is employed to improve the convergence rate of signals in the closed-loop system.

Keywords
adaptive control, disturbance compensation, Falb–Wolovich approach, harmonic disturbance, robust control

For citation: Tu C.V., Dudarenko N.A. Robust adaptive compensation of unknown multi-harmonic disturbances in multivariable linear systems. *Scientific and Technical Journal of Information Technologies, Mechanics and Optics*, 2026, vol. 26, no. 5, pp. 935–944. doi: 10.17586/2226-1494-2026-26-5-935-944

УДК 618.51

Робастная адаптивная компенсация неизвестных мультигармонических возмущений в многомерных линейных системах

Конг Винь Ты[1]✉, Наталия А. Дударенко[2]

[1,2] Университет ИТМО, Санкт-Петербург, 197101, Российская Федерация

[1] congvinhvkdn@gmail.com✉, https://orcid.org/0009-0005-5106-8560
[2] dudarenko@itmo.ru, https://orcid.org/0000-0002-3553-0584

Аннотация
Предлагается новый метод прямой адаптивной компенсации возмущений для класса многомерных линейных систем, обладающий свойством робастности по отношению к внешним возмущениям. Предполагается, что внешние возмущения представляют собой мультигармонические сигналы (регулярная составляющая) с добавочным случайным шумом (нерегулярная составляющая), при этом смещение, фаза, амплитуда и частота являются неизвестными. Представлена разработка закона управления с обратной связью по состоянию, обеспечивающего сходимость вектора выходной переменной к нулю при отсутствии нерегулярной составляющей возмущения. Робастность замкнутой системы по отношению к аддитивным немоделируемым возмущениям проявляется в том, что предлагаемый закон управления гарантирует ограниченность всех сигналов замкнутой системы. Решение задачи базируется на использовании метода развязки каналов системы Фальба–Воловича с обратной связью по состоянию с последующим применением метода прямой компенсации возмущений,

© Tu C.V., Dudarenko N.A., 2026

основанного на принципе внутренней модели. В результате робастный адаптивный алгоритм строится независимо для каждого канала управления. Для повышения скорости сходимости сигналов в замкнутой системе используется адаптивный алгоритм с расширением памяти регрессора.

Ключевые слова
адаптивное управление, гармонические возмущения, компенсация возмущений, подход Фальба–Воловича, робастное управление

Ссылка для цитирования: Ты К.В., Дударенко Н.А. Робастная адаптивная компенсация неизвестных мультигармонических возмущений в многомерных линейных системах // Научно-технический вестник информационных технологий, механики и оптики. 2026. Т. 26, № 5. С. 935–944 (на англ. яз.). doi: 10.17586/2226-1494-2026-26-5-935-944

Introduction

The paper focuses on the problem of robust adaptive disturbance compensation for a class of Multi-Input Multi-Output (MIMO) Linear Time-Invariant (LTI) systems. The external disturbances considered in this study are unknown and harmonic in nature. An effective approach to disturbance compensation is based on the Internal Model Principle (IMP). According to this approach, the disturbance is modeled as the output of a linear autonomous system, and consequently, the states and parameters of this system must be incorporated into the control law to achieve zero steady-state error. Adaptive disturbance compensation based on the IMP has attracted considerable attention in the literature, particularly when disturbances are represented as outputs of exosystems with unknown parameters (see Section 1.4 of [1, 2]). The main objective of these approaches is to design adaptive control laws based on either state feedback (see Section 4.2 of [1, 3, 4]) or output feedback [2, 5]. These methods have been successfully applied to both Single-Input Single-Output (SISO) [6–10] and MIMO systems (see Section 4.2 of [1]).

According to many previous studies, adaptive algorithms generally exhibit good convergence performance under ideal conditions, namely in the absence of disturbances and with constant system parameters. However, in the presence of bounded disturbances, measurement noise, and time-varying parameters, the stability of recursive estimation algorithms cannot be guaranteed without introducing modification mechanisms such as dead-zones, leakage terms (σ-modifications), or parameter projection techniques [11]. Thus, the adaptive disturbance compensation algorithm is designed under the assumption that unmodeled disturbances are absent. However, unmodeled disturbances may destroy the stability of the closed-loop adaptive system [12–14]. In particular, even an asymptotically vanishing unmodeled disturbance can cause unbounded parametric drift.

The problem of robust adaptive disturbance compensation for systems with unmatched disturbances has been studied in the literature [1] but only for SISO linear systems. Therefore, this paper proposes a new method to address this problem for MIMO systems. The proposed approach begins with the decoupling of the system inputs and outputs using Falb–Wolovich approach. Then, the direct disturbance compensation method based on the IMP is applied to the resulting decoupled system. As a result, the adaptive algorithm can be constructed robustly and independently for each control channel. The proposed method guarantees that the adaptive algorithm is capable of regulating the system output to zero in the absence of unmodeled disturbances. Moreover, in the presence of unmodeled disturbances, the adaptive algorithm guarantees boundedness of all signals in the closed-loop system.

Problem statement

Let us consider the system

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{D}\mathbf{v}_f(t), \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{E}\mathbf{v}_f(t), \end{cases} \quad (1)$$

where $\mathbf{x}(t) \in \mathbb{R}^n$, $\mathbf{u}(t) \in \mathbb{R}^m$, $\mathbf{y}(t) \in \mathbb{R}^m$ and $\mathbf{v}_f(t) \in \mathbb{R}^k$ denote the state vector, control input vector, output vector, and external disturbance vector, respectively. The matrices $\mathbf{A} \in \mathbb{R}^{n\times n}$, $\mathbf{B} \in \mathbb{R}^{n\times m}$, $\mathbf{C} \in \mathbb{R}^{m\times n}$ and $\mathbf{D} \in \mathbb{R}^{n\times k}$ are known, while matrix $\mathbf{A}$ may be unstable, the matrix $\mathbf{E} \in \mathbb{R}^{m\times k}$ may be unknown, and the dimensions satisfy $m \le n$.

For this paper, the following assumptions are considered.

Assumption 1. The pair of matrices ($\mathbf{A}$, $\mathbf{B}$) is controllable, while the pair ($\mathbf{A}$, $\mathbf{C}$) is observable.

Assumption 2. The external disturbance $\mathbf{v}_f(t) = [v_{f1}(t), v_{f2}(t), \ldots, v_{fk}(t)]^\top$ is bounded and unmeasurable. Each component of the disturbance $v_{f\alpha}(t)$ ($\alpha = 1, 2, \ldots, k$) can be represented as

$$v_{f\alpha}(t) = v_\alpha(t) + f_\alpha(t), \quad (2)$$

where $v_\alpha(t)$ is the deterministic component and $f_\alpha(t)$ is the irregular component.

The irregular component of the disturbance (2) is bounded and unknown, $|f_\alpha(t)| \le \bar{f}_\alpha$, where $\bar{f}_\alpha$ is an unknown constant.

The deterministic component of the disturbance $v_\alpha(t)$ is generated by a linear autonomous system:

$$\dot{\xi}_{v\alpha}(t) = \mathbf{\Gamma}_{v\alpha}\xi_{v\alpha}(t),\ v_\alpha(t) = \mathbf{h}_{v\alpha}^\top\xi_{v\alpha}(t). \quad (3)$$

Here, the state vector $\xi_{v\alpha}(t) \in \mathbb{R}^{q_\alpha}$ is not directly measurable. The matrix $\mathbf{\Gamma}_{v\alpha}$ is constant and unknown, with simple eigenvalues located on the imaginary axis. The vector $\mathbf{h}_{v\alpha}$ is also unknown but constant. Each pair ($\mathbf{\Gamma}_{v\alpha}$, $\mathbf{h}_{v\alpha}$) is observable, and the upper bound q_α is known.

Remark 1. According to Assumption 2, the external disturbance can be represented in the form $\mathbf{v}_f(t) = \mathbf{v}(t) + \mathbf{f}(t)$, where $\mathbf{v}(t)$ is the deterministic component and $\mathbf{f}(t)$ is the irregular component. Then, the deterministic component $\mathbf{v}(t)$ can be represented in the simplified multi-output system form as follows:

$$\dot{\xi}_v(t) = \mathbf{\Gamma}_v \xi_v(t),\ \mathbf{v}(t) = \mathbf{H}_v^\top \xi_v(t),$$

where $\mathbf{\Gamma}_v = \text{blkdiag}\{\mathbf{\Gamma}_{v\alpha}\}$, $\mathbf{H}_v^\top = \text{blkdiag}\{\mathbf{h}_{v\alpha}^\top\}$, $\xi_v(t) = [\xi_{v1}^\top(t), \ldots, \xi_{v\alpha}^\top(t)]^\top$. Here, blkdiag$\{*\}$ is the block-diagonal matrix with diagonal elements $\{*\}$.

The control objective is to design a control algorithm that guarantees boundedness of all signals in the closed-loop system while satisfying the following conditions:

1. when $\mathbf{f}(t) \neq 0$, the output $\mathbf{y}(t)$ converges to a compact residual set:
$$\|\mathbf{y}(t)\| \leq D,\ t \to \infty;$$
2. when $\mathbf{f} \equiv 0$, the following condition is satisfied:
$$\lim_{t\to\infty} \|\mathbf{y}(t)\| = 0.$$

Here, $\|(*)\|$ is the Euclidean norm of a vector $(*)$.

Design of the decoupler

The state-feedback decoupling controller based on Falb–Wolovich approach [15] is designed to decouple the control inputs and outputs of the system described in (1). It has the following structure:

$$\mathbf{u}(t) = -\mathbf{K}\mathbf{x}(t) + \mathbf{F}\mathbf{u}_d(t), \tag{4}$$

where $\mathbf{u}_d(t) \in \mathbb{R}^m$ is the control input for the system obtained after decoupling; $\mathbf{K}$ and $\mathbf{F}$ are constant real-valued matrices with compatible dimensions.

Let the output matrix $\mathbf{C}$ can be written as $\mathbf{C} = [\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_m]^\top$, where c_i represents the i-th row of the matrix $\mathbf{C}$. For every $i = 1, 2, \ldots, m$, we define a positive integer ρ_i, known as the relative degree, as follows:

$$\rho_i = \begin{cases} \min(l | \mathbf{c}_i\mathbf{A}^{l-1}\mathbf{B} \neq \mathbf{0}^\top),\ l = 1, 2, \ldots, n-1, \\ n-1, \text{ if } \mathbf{c}_i\mathbf{A}^{l-1}\mathbf{B} = \mathbf{0}^\top,\ l = 1, 2, \ldots, n. \end{cases} \tag{5}$$

Here, $\mathbf{0}^\top \in \mathbb{R}^m$ denotes a row vector whose entries are all zero.

To ensure that the decoupled system possesses the desired pre-assigned poles, the control gain matrices $\mathbf{K}$ and $\mathbf{F}$ are designed accordingly. Let $\overline{n} = \sum_{i=1}^{m} \rho_i$ denote the total number of desired poles to be assigned to the feedback system. For each channel $i = 1, 2, \ldots, m$, let $\lambda_{ij}\ j = 1, 2, \ldots, \rho_i$, denote the desired poles assigned to that channel. We then define the corresponding monic polynomial as

$$\mathcal{L}_i(s) = \prod_{j=1}^{\rho_i} (s - \lambda_{ij}),\ i = 1, 2, \ldots, m.$$

For each i, the polynomial $\mathcal{L}_i(s)$ can be expressed as

$$\mathcal{L}_i(s) = q_{i0} + q_{i1}s + q_{i2}s^2 + \cdots + s^{\rho_i}.$$

Here, s denotes the Laplace variable, and q_{iz}, $z = 0, 1, \ldots, \rho_i - 1$, are constant coefficients determined by the assigned poles λ_{ij}. In particular, q_{i0} is the constant term of the monic polynomial $\mathcal{L}_i(s)$. The term s^{ρ_i} is its highest-order term, where ρ_i is the relative degree defined in (5).

Therefore, the desired transfer function between the control input and the output of the i-th decoupled channel is defined as

$$G_{ii}(s) = \frac{q_{i0}}{q_{i0} + q_{i1}s + q_{i2}s^2 + \cdots + s^{\rho_i}},\ i = 1, 2, \ldots, m.$$

Here, $G_{ii}(s)$ denotes the desired transfer function of the i-th decoupled channel.

Let us define the matrices $\mathbf{B}^*$ and $\mathbf{C}^*$ as follows:

$$\mathbf{B}^* = \begin{bmatrix} \mathbf{c}_1\mathbf{A}^{\rho_1-1}\mathbf{B} \\ \mathbf{c}_2\mathbf{A}^{\rho_2-1}\mathbf{B} \\ \vdots \\ \mathbf{c}_m\mathbf{A}^{\rho_m-1}\mathbf{B} \end{bmatrix},\quad \mathbf{C}^* = \begin{bmatrix} \mathbf{c}_1\mathbf{A}^{\rho_1} + \sum_{z=0}^{\rho_1-1} q_{1z}\mathbf{c}_1\mathbf{A}^z \\ \vdots \\ \mathbf{c}_m\mathbf{A}^{\rho_m} + \sum_{z=0}^{\rho_m-1} q_{mz}\mathbf{c}_m\mathbf{A}^z \end{bmatrix},$$

where $\mathbf{A}^z$ denotes the z-th power of the system matrix $\mathbf{A}$.

Based on these definitions, we can formulate the following result.

Proposition 1. The system described in (1) can be decoupled if and only if the matrix $\mathbf{B}^*$ is invertible. Under this condition, the gain matrices in the control expression (4) are determined as

$$\mathbf{K} = (\mathbf{B}^*)^{-1}\mathbf{C}^*,\ \mathbf{F} = (\mathbf{B}^*)^{-1}\text{diag}\{q_{i0}\}.$$

Consequently, the overall transfer function matrix of the system becomes diagonal and can be written as

$$\mathbf{G}_u(s) = \mathbf{C}(s\mathbf{I} - (\mathbf{A} - \mathbf{B}\mathbf{K}))^{-1}\mathbf{B}\mathbf{F} = \text{diag}\{G_{ii}(s)\}.$$

Here, diag$\{*\}$ is the diagonal matrix with diagonal elements $\{*\}$, $\mathbf{I}$ is identity matrix of dimension $n \times n$.

For the proof see [16, Chapter 4].

Remark 2. Following the decoupling of system (1) via the control strategy defined in (4) using the matrices $\mathbf{K}$ and $\mathbf{F}$ from Proposition 1, the overall system can then be expressed in the state-space form:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}_\Sigma\mathbf{x}(t) + \mathbf{B}_\Sigma\mathbf{u}_d(t) + \mathbf{D}\mathbf{v}_f(t), \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{E}\mathbf{v}_f(t), \end{cases} \tag{6}$$

where the matrices $\mathbf{A}_\Sigma = \mathbf{A} - \mathbf{B}\mathbf{K}$ and $\mathbf{B}_\Sigma = \mathbf{B}\mathbf{F}$ describe the decoupled dynamics. The transfer matrix $\mathbf{G}_u(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A}_\Sigma)^{-1}\mathbf{B}_\Sigma$ has a diagonal form, and $\mathbf{A}_\Sigma$ is Hurwitz.

Design of the disturbance observer

The external disturbance can also be represented in its canonical form [1, 17, 18]. The disturbance observer will be constructed based on this disturbance representation.

The exosystem described in (3) can be transformed into the following canonical form:

$$\dot{\bar{\xi}}_{v\alpha}(t) = \mathbf{G}_{v\alpha}\bar{\xi}_{v\alpha}(t) + \mathbf{I}_{v\alpha}v_\alpha(t),\ v_\alpha(t) = \boldsymbol{\theta}_{v\alpha}^\top\bar{\xi}_{v\alpha}(t), \tag{7}$$

where $\bar{\xi}_{v\alpha}(t) \in \mathbb{R}^{q_\alpha}$ (for $\alpha = 1, 2, \ldots, k$) denotes the state vector with initial value $\bar{\xi}_{v\alpha}(0)$; $\boldsymbol{\theta}_{v\alpha} \in \mathbb{R}^{q_\alpha}$ represents a parameter vector that is unknown; $\mathbf{G}_{v\alpha} \in \mathbb{R}^{q_\alpha \times q_\alpha}$ is a Hurwitz matrix; and $\mathbf{I}_{v\alpha}$ is a constant vector ensuring the controllability of the pair $(\mathbf{G}_{v\alpha}, \mathbf{I}_{v\alpha})$.

The exosystem given in (7) can alternatively be written in a block-diagonal structure as:

$$\dot{\bar{\xi}}_v(t) = \mathbf{G}_v\bar{\xi}_v(t) + \mathbf{L}_v\mathbf{v}(t),\ \mathbf{v}(t) = \boldsymbol{\theta}_v^\top\bar{\xi}_v(t), \tag{8}$$

or equivalently:

$$\dot{\bar{\xi}}_v(t) = (\mathbf{G}_v + \mathbf{L}_v\boldsymbol{\theta}_v^\top)\bar{\xi}_v(t),\ \mathbf{v}(t) = \boldsymbol{\theta}_v^\top\bar{\xi}_v(t), \tag{9}$$

where $\bar{\xi}_v(t) = [\bar{\xi}_{v1}^\top(t),\ \bar{\xi}_{v2}^\top(t)\ \ldots,\ \bar{\xi}_{vk}^\top(t)]^\top \in \mathbb{R}^q$; $\boldsymbol{\theta}_v^\top = \text{blkdiag}\{\boldsymbol{\theta}_{v\alpha}^\top\} \in \mathbb{R}^{k\times q}$; $\mathbf{G}_v = \text{blkdiag}\{\mathbf{G}_{v\alpha}\} \in \mathbb{R}^{q\times q}$; $\mathbf{L}_v = \text{blkdiag}\{\mathbf{I}_{v\alpha}\} \in \mathbb{R}^{q\times k}$, and $q = \sum_{\alpha=1}^{k} q_\alpha$.

Following equation (8), the disturbance observer proposed in [19] is constructed to estimate the state $\bar{\xi}_v(t)$ in the following structure:

$$\begin{cases} \dot{\boldsymbol{\phi}}(t) = \mathbf{G}_v\boldsymbol{\phi}(t) + (\mathbf{G}_v\mathbf{N} - \mathbf{N}\mathbf{A}_\Sigma)\mathbf{x}(t) - \mathbf{N}\mathbf{B}_\Sigma\mathbf{u}_d(t), \\ \hat{\bar{\xi}}_v(t) = \boldsymbol{\phi}(t) + \mathbf{N}\mathbf{x}(t), \end{cases} \tag{10}$$

where the initial value $\boldsymbol{\phi}(0)$ is arbitrary. The matrix $\mathbf{N} \in \mathbb{R}^{q\times n}$ is selected such that it satisfies the condition

$$\mathbf{N}\mathbf{D} = \mathbf{L}_v. \tag{11}$$

The state estimation error $\mathbf{e}_v$ of the canonical form (8) in the presence of the irregular component is defined as follows:

$$\mathbf{e}_v = \bar{\xi}_v(t) - \hat{\bar{\xi}}_v(t), \tag{12}$$

where $\hat{\bar{\xi}}_v$ is generated by the observer (10) and (11) in the presence of the irregular component.

Definition 1. An observer is said to be robust if the regressor estimation error $\mathbf{e}_v$ generated by the observer satisfies the following conditions:

1. if $\mathbf{f}(t) \neq 0$, then $\mathbf{e}_v$ remains bounded;
2. if $\mathbf{f}(t) \to 0$ as $t \to \infty$, then

$$\|\mathbf{e}_{v(t)}\| \to 0 \text{ as } t \to \infty;$$

3. if $\mathbf{f} \equiv 0$, then the estimation error $\mathbf{e}_v(t)$ converges to zero exponentially.

Definition 2. The regressor estimation error $\mathbf{e}_v$ is said to be robust in the sense of $\mathbf{f}$ if it satisfies properties 1–3 stated in Definition 1.

Lemma 1. If Assumption 2 is satisfied and there exists a matrix $\mathbf{N}$ satisfying (11), then the state $\bar{\xi}_v$ of the exosystem canonical form (8) can be expressed as the sum of its observer-based estimate and the corresponding estimation error: $\bar{\xi}_v = \hat{\bar{\xi}}_v + \mathbf{e}_v$. Here, $\hat{\bar{\xi}}_v$ is the bounded estimate generated by the observer (10) and (11), and $\mathbf{e}_v$ is robust according to Definition 2.

Proof. Differentiating the expression (12) with respect to time and taking into account the equations (6), (8), (10), (11), we obtain

$$\dot{\mathbf{e}}_v = \mathbf{G}_v\dot{\bar{\xi}}_v + \mathbf{L}_v\mathbf{v} - \mathbf{G}_v\boldsymbol{\phi} - \mathbf{G}_v\mathbf{N}\mathbf{x} + \mathbf{N}\mathbf{A}_\Sigma\mathbf{x} + \mathbf{N}\mathbf{B}_\Sigma\mathbf{u}_d - \\ - \mathbf{N}\mathbf{A}_\Sigma\mathbf{x} - \mathbf{N}\mathbf{B}_\Sigma\mathbf{u}_d - \mathbf{N}\mathbf{D}\mathbf{v} - \mathbf{N}\mathbf{D}\mathbf{f} = \mathbf{G}_v(\bar{\xi}_v - \boldsymbol{\phi} - \mathbf{N}\mathbf{x}) - \mathbf{L}_v\mathbf{f} = \\ = \mathbf{G}_v\mathbf{e}_v - \mathbf{L}_v\mathbf{f}.$$

Since $\mathbf{G}_v$ is Hurwitz, $\mathbf{e}_v$ satisfies the properties stated in Definition 2. ■

Synthesis of the control law and adaptation algorithm

The decoupling process transforms system (1) into the form described in (6), in which the disturbances are unmatched with the control input. Therefore, the matrix regulator equation [1, 20, 21] is employed to transform system (6) into a form where the external disturbances are compatible with the control input.

Under Assumptions 1 and 2, for system (6), the following results are valid [1]: There exist matrices $\mathbf{M}_v \in \mathbb{R}^{n\times q}$, $\boldsymbol{\varphi} \in \mathbb{R}^{q\times k}$ such that the matrix equalities hold:

$$\begin{gathered} \mathbf{B}_\Sigma\boldsymbol{\varphi}^\top = \mathbf{A}_\Sigma\mathbf{M}_v - \mathbf{M}_v(\mathbf{G}_v + \mathbf{L}_v\boldsymbol{\theta}_v^\top) + \mathbf{D}\boldsymbol{\theta}_v^\top, \\ \mathbf{C}\mathbf{M}_v = -\mathbf{E}\boldsymbol{\theta}_v^\top. \end{gathered} \tag{13}$$

It should be highlighted that the transmission zeros of system (1), which are not located in the minimum phase region, are effectively removed through the decoupling procedure. According to Assumptions 2, the eigenvalues of the matrix $\boldsymbol{\Gamma}_v$ (or, equivalently, $\mathbf{G}_v + \mathbf{L}_v\boldsymbol{\theta}_v^\top$) are ensured not to coincide with the transmission zeros of the transfer matrix $\mathbf{G}_u(s)$.

We define the state error as $\mathbf{e}(t) = \mathbf{x}(t) - \mathbf{M}\bar{\xi}_v(t)$. Then, taking into account equations (6), (9), we compute the time derivative of $\mathbf{e}(t)$ and obtain:

$$\dot{\mathbf{e}}(t) = \mathbf{A}_\Sigma\mathbf{e}(t) + \mathbf{B}_\Sigma\mathbf{u}_d(t) + (\mathbf{A}_\Sigma\mathbf{M}_v - \mathbf{M}_v(\mathbf{G}_v + \mathbf{L}_v\boldsymbol{\theta}_v^\top) + \\ + \mathbf{D}\boldsymbol{\theta}_v^\top)\bar{\xi}_v(t).$$

Based on equations (12) and (13), the system can be rewritten as follows:

$$\begin{cases} \dot{\mathbf{e}}(t) = \mathbf{A}_\Sigma\mathbf{e}(t) + \mathbf{B}_\Sigma(\mathbf{u}_d(t) + \boldsymbol{\varphi}^\top\hat{\bar{\xi}}(t) + \mathbf{f}_e(t)), \\ \mathbf{y}(t) = \mathbf{C}\mathbf{e}(t) + \mathbf{E}\mathbf{f}(t), \end{cases} \tag{14}$$

where $\mathbf{f}_e = \boldsymbol{\varphi}^\top\mathbf{e}_v$ is robust in the sense of Definition 2.

Since the matrix $\mathbf{A}_\Sigma$ is Hurwitz, the control law for system (14) can be formulated as follows:

$$\mathbf{u}_d(t) = -\hat{\boldsymbol{\varphi}}^\top\hat{\bar{\xi}}(t). \tag{15}$$

Here, the matrix $\hat{\boldsymbol{\varphi}}^\top = [\hat{\boldsymbol{\varphi}}_1^\top(t),\ \hat{\boldsymbol{\varphi}}_2^\top(t),\ \ldots,\ \hat{\boldsymbol{\varphi}}_m^\top(t)]^\top \in \mathbb{R}^{m\times q}$ is determined using an adaptive algorithm.

As a result, the system described in equation (14) can be reformulated as

$$\mathbf{y}(t) = \mathbf{G}_u(s)[\tilde{\boldsymbol{\varphi}}^\top\hat{\bar{\xi}}(t)] + \mathbf{f}_y, \tag{16}$$

where $\tilde{\boldsymbol{\varphi}}^\top = \boldsymbol{\varphi}^\top - \hat{\boldsymbol{\varphi}}^\top$ is the matrix identification error of the adjustable parameters, and

$$\mathbf{f}_y = \mathbf{G}_u(s)[\mathbf{f}_e] + \mathbf{C}e^{\mathbf{A}_\Sigma t}\mathbf{e}(0) + \mathbf{E}\mathbf{f}, \tag{17}$$

is a bounded disturbance.

From expression (16), the augmented error is constructed as follows:

$$\bar{\mathbf{y}} = \mathbf{y} + \mathbf{G}_u(s)[\hat{\boldsymbol{\varphi}}^\top\hat{\bar{\xi}}] - \boldsymbol{\Delta}^\top\hat{\boldsymbol{\varphi}}, \tag{18}$$

where

$$\boldsymbol{\Delta}^\top = \mathbf{G}_u(s)[\hat{\bar{\xi}}^\top] = \begin{bmatrix} G_{11}(s)[\hat{\bar{\xi}}^\top] & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & G_{mm}(s)[\hat{\bar{\xi}}^\top] \end{bmatrix}.$$

Then, the output error model takes the following form:

$$\bar{\mathbf{y}} = \boldsymbol{\Delta}^{\top}\tilde{\boldsymbol{\varphi}} + \mathbf{f}_y. \tag{19}$$

Remark 3. Due to the diagonal structure of $\mathbf{G}_u$, the adaptive control law (15) can be designed independently for each channel. However, since the internal realization matrix $\mathbf{A}_\Sigma$ is not block diagonal, residual interactions between channels still remain in the closed-loop dynamics. These interactions are represented by the bounded disturbance terms f_{yi}, which include inter-channel coupling effects. Therefore, each channel can be represented by the scalar error model

$$\bar{y}_i = \tilde{\boldsymbol{\varphi}}_i^{\top}\boldsymbol{\Delta}_i + f_{yi}, \tag{20}$$

where $\boldsymbol{\Delta}_i = G_{ii}(s)[\hat{\boldsymbol{\xi}}]$, and the term f_{y_i} is bounded.

A. Gradient-based algorithm of adaptation

Because the regression matrix $\boldsymbol{\Delta}^{\top}$ has a block-diagonal structure, in the case $\mathbf{f}(t) \equiv 0$, the gradient-based adaptation algorithm can be independently constructed for each control channel as follows [1]:

$$\dot{\hat{\boldsymbol{\varphi}}}_i = \gamma_i \boldsymbol{\Delta}_i \bar{y}_i,\ i = 1, 2, \ldots, m, \tag{21}$$

where γ_i is the adaptation gain.

Therefore, the following theorem is formulated and proved by analogy with the case of SISO systems presented in [1].

Theorem 1. The robust gradient algorithm of adaptation (21) is constructed as follows:

$$\dot{\hat{\boldsymbol{\varphi}}}_i = \gamma_i \boldsymbol{\Delta}_i \bar{y}_i - \gamma_i \sigma(\hat{\boldsymbol{\varphi}}_i)\hat{\boldsymbol{\varphi}}_i,\ i = 1, 2, \ldots, m, \tag{22}$$

where

$$\sigma(\hat{\boldsymbol{\varphi}}_i) = \begin{cases} 0, & \|\hat{\boldsymbol{\varphi}}_i\| < \varphi_i^*, \\ \dfrac{\|\hat{\boldsymbol{\varphi}}_i\|}{\varphi_i^*} - 1, & \varphi_i^* \le \|\hat{\boldsymbol{\varphi}}_i\| \le 2\varphi_i^*, \\ 1 & \|\hat{\boldsymbol{\varphi}}_i\| > 2\varphi_i^*, \end{cases} \tag{23}$$

and φ_i^* is a sufficiently large positive constant, ensures that:

T.1.1. All signals in the closed-loop system are bounded, and if $\varphi_i^* \ge \|\boldsymbol{\varphi}_i\|$, then the output vector norm $\|\mathbf{y}\|$ is of the order of $\|\bar{\mathbf{f}}\| > \|\mathbf{f}(t)\|$ in the mean-square sense, i.e., $\|\mathbf{y}\| \in \mathcal{MS}(\|\bar{\mathbf{f}}\|)$.

T.1.2. If $\varphi_i^* \ge \|\boldsymbol{\varphi}_i\|$, and $\mathbf{f} \equiv 0$, then the output signal asymptotically converges to zero, i.e., $\mathbf{y}(t) \to 0$ as $t \to \infty$.

Proof. From expression (22), in view of the equality $\dot{\hat{\boldsymbol{\varphi}}}_i = -\dot{\tilde{\boldsymbol{\varphi}}}_i$ the model of the parametric errors can be represented as

$$\dot{\tilde{\boldsymbol{\varphi}}}_i = -\gamma_i \boldsymbol{\Delta}_i \bar{y}_i + \gamma_i \sigma(\hat{\boldsymbol{\varphi}}_i)\hat{\boldsymbol{\varphi}}_i. \tag{24}$$

Choose the Lyapunov function as $\mathbf{V} = \dfrac{1}{2\gamma_i}\tilde{\boldsymbol{\varphi}}_i^{\top}\tilde{\boldsymbol{\varphi}}_i$. Taking the time derivative yields:

$$\begin{aligned} \dot{\mathbf{V}} &= -\tilde{\boldsymbol{\varphi}}_i^{\top}\boldsymbol{\Delta}_i\bar{y}_i + \sigma(\hat{\boldsymbol{\varphi}}_i)\tilde{\boldsymbol{\varphi}}_i^{\top}\hat{\boldsymbol{\varphi}}_i = -\bar{y}_i^2 + \bar{y}_i f_{yi} + \sigma(\hat{\boldsymbol{\varphi}}_i)\tilde{\boldsymbol{\varphi}}_i^{\top}\hat{\boldsymbol{\varphi}}_i \le \\ &\le -\bar{y}_i^2 + \bar{f}_{yi}|\bar{y}_i| + \sigma\tilde{\boldsymbol{\varphi}}_i^{\top}\hat{\boldsymbol{\varphi}}_i \le \\ &\le -\frac{1}{2}\bar{y}_i^2 - \frac{1}{2}(\bar{y}_i - \bar{f}_{yi})^2 + \frac{1}{2}\bar{f}_{yi}^2 + \sigma\tilde{\boldsymbol{\varphi}}_i^{\top}\hat{\boldsymbol{\varphi}}_i \le \\ &\le -\frac{1}{2}\bar{y}_i^2 + \frac{1}{2}\bar{f}_{yi}^2 + \sigma\tilde{\boldsymbol{\varphi}}_i^{\top}\hat{\boldsymbol{\varphi}}_i, \end{aligned} \tag{25}$$

where $\bar{f}_{yi}$ is the bound of $|f_{yi}|$.

Because $\sigma\tilde{\boldsymbol{\varphi}}_i^{\top}\hat{\boldsymbol{\varphi}}_i = -\sigma\|\tilde{\boldsymbol{\varphi}}_i\|^2 + \sigma\tilde{\boldsymbol{\varphi}}_i^{\top}\boldsymbol{\varphi}_i$, the inequality can be rewritten in the form

$$\dot{V} \le -\frac{1}{2}\bar{y}_i^2 - \sigma\|\tilde{\boldsymbol{\varphi}}_i\|^2 + \frac{1}{2}\bar{f}_{yi}^2 + \sigma\tilde{\boldsymbol{\varphi}}_i^{\top}\boldsymbol{\varphi}_i.$$

The following inequality is valid:

$$-\sigma(\hat{\boldsymbol{\varphi}}_i)\|\tilde{\boldsymbol{\varphi}}_i\|^2 \le -k_1\|\tilde{\boldsymbol{\varphi}}_i\|^2 + k_2,$$

where $0 < k_1 < 1$ and $k_2 = k_1(\|\boldsymbol{\varphi}_i\| + (1 + k_1)\varphi_i^*)^2$. Then

$$\begin{aligned} \dot{V} &\le -\frac{1}{2}\bar{y}_i^2 - k_1\|\tilde{\boldsymbol{\varphi}}_i\|^2 + \|\tilde{\boldsymbol{\varphi}}_i\|\,\|\boldsymbol{\varphi}_i\| + \frac{1}{2}\bar{f}_{yi}^2 + k_2 \le \\ &\le -\frac{1}{2}\bar{y}_i^2 - \frac{k_1}{2}\|\tilde{\boldsymbol{\varphi}}_i\|^2 - \left(\sqrt{\frac{k_1}{2}}\,\|\tilde{\boldsymbol{\varphi}}_i\| - \frac{1}{\sqrt{2k_1}}\,\|\boldsymbol{\varphi}_i\|\right)^2 + \\ &\quad + \frac{1}{2k_1}\|\boldsymbol{\varphi}_i\|^2 + \frac{1}{2}\bar{f}_{yi}^2 + k_2 \le \\ &\le -\frac{1}{2}\bar{y}_i^2 - \frac{k_1}{2}\|\tilde{\boldsymbol{\varphi}}_i\|^2 + \frac{1}{2k_1}\|\boldsymbol{\varphi}_i\|^2 + \frac{1}{2}\bar{f}_{yi}^2 + k_2 \le \\ &\le -k_1\gamma_i V + \frac{1}{2k_1}\|\boldsymbol{\varphi}_i\|^2 + \frac{1}{2}\bar{f}_{yi}^2 + k_2. \end{aligned} \tag{26}$$

The final inequality (26) proves that $\hat{\boldsymbol{\varphi}}_i$ is bounded and exponentially converges to the residual set D_i defined as follows:

$$D_i = \left\{\tilde{\boldsymbol{\varphi}}_i:\ \|\tilde{\boldsymbol{\varphi}}_i\|^2 \le \frac{1}{k_1}\left(\frac{1}{k_1}\|\boldsymbol{\varphi}_i\|^2 + \bar{f}_{yi}^2 + 2k_2\right)\right\}. \tag{27}$$

Since $\boldsymbol{\Delta}_i$ is bounded, from the boundedness of $\hat{\boldsymbol{\varphi}}_i$ due to (27) and f_{yi}, we can directly conclude that $\bar{y}_i$ and $\|\dot{\hat{\boldsymbol{\varphi}}}_i\|$ are also bounded.

Integrating inequality (25) over the interval $[t, t + T]$, we obtain [22]:

$$-2\int_t^{t+T}\sigma\tilde{\boldsymbol{\varphi}}_i^{\top}\hat{\boldsymbol{\varphi}}_i d\tau + \int_t^{t+T}\bar{y}_i^2 d\tau \le \bar{f}_{yi}^2 T + k_3, \tag{28}$$

where

$$k_3 = 2\sup_{t\ge 0}(V(t) - V(t+T)),\ T \ge 0,$$

and τ is variable of integration.

Since $\varphi_i^* \ge \|\boldsymbol{\varphi}_i\|$, the following inequality holds: $\sigma\tilde{\boldsymbol{\varphi}}_i^{\top}\hat{\boldsymbol{\varphi}}_i \le 0$. Therefore, from (28), it follows that $\bar{y}_i \in \mathcal{MS}(\bar{\mathbf{f}}_{yi})$. Furthermore, because $\boldsymbol{\Delta}_i$ is bounded, relation (22) implies that $\|\dot{\hat{\boldsymbol{\varphi}}}_i\|,\ \bar{y}_i \in \mathcal{MS}(\bar{\mathbf{f}}_{yi})$. Therefore, $\|\bar{\mathbf{y}}\| \in \mathcal{MS}\|\bar{\mathbf{f}}_y\|$. From (17), we obtain that $\|\mathbf{f}_y\| \in \mathcal{MS}(\|\bar{\mathbf{f}}\|)$. As a result, $\|\bar{\mathbf{y}}\| \in \mathcal{MS}(\|\bar{\mathbf{f}}\|)$. From (18), we conclude that the difference between $\mathbf{y}$ and $\bar{\mathbf{y}}$ is of the same order as $\mathbf{f}_y$ [1]; hence $\|\mathbf{y}\| \in \mathcal{MS}(\|\bar{\mathbf{f}}\|)$, and Property T.1.1 is proved. Here, $\mathcal{MS}(*)$ is the space of time functions that are of the order of $*$ in the mean-square sense.

To establish Property T.1.2 [1], consider the Lyapunov-like functional

$$V(\tilde{\boldsymbol{\varphi}}_i) = \frac{1}{2}\tilde{\boldsymbol{\varphi}}_i^{\top}\tilde{\boldsymbol{\varphi}}_i + \int_t^{\infty} f_{yi}^2(\tau)d\tau.$$

Here, the integral term is finite because f_{yi} exponentially decays in the case $\mathbf{f} \equiv 0$, as follows from expression (17). Taking the time derivative of V along the solutions of (24), we obtain

$$\dot{V} = -\bar{y}_i^2 + \bar{y}_i f_{yi} - f_{yi}^2 + \sigma\tilde{\boldsymbol{\varphi}}_i^\top \hat{\boldsymbol{\varphi}}_i. \quad (29)$$

Since $\varphi_i^* \geq ||\boldsymbol{\varphi}_i||$, the following inequality holds: $\sigma\tilde{\boldsymbol{\varphi}}_i^\top \hat{\boldsymbol{\varphi}}_i \leq 0$, for any $\hat{\boldsymbol{\varphi}}_i$. Therefore, time derivative of V (29) satisfies the following inequality

$$\dot{V} \leq -\bar{y}_i^2 + \bar{y}_i f_{yi} - f_{yi}^2 = -\frac{1}{2}\bar{y}_i^2 - \frac{1}{2}(\bar{y}_i - f_{yi})^2 -$$

$$-\frac{1}{2}f_{yi}^2 \leq -\frac{1}{2}\bar{y}_i^2. \quad (30)$$

From (30), we have

$$\int_0^\infty \bar{y}_i^2(\tau)d\tau \leq -2\int_0^\infty \dot{V}(\tau)d\tau,$$

where $\dot{V}(\tau)$ is derivative of V at time τ.

Since $V(t)$ is non-increasing and bounded from below, the limit $V_\infty = \lim_{t\to\infty} V(t)$ exists and is finite. Therefore,

$$\int_0^\infty \dot{V}(\tau)d\tau = V_\infty - V(0).$$

Thus,

$$\int_0^\infty \bar{y}_i^2(\tau)d\tau \leq -2(V_\infty - V(0)) < \infty.$$

From (20), we have

$$\dot{\bar{y}}_i = \dot{\tilde{\boldsymbol{\varphi}}}_i^\top \boldsymbol{\Delta}_i + \tilde{\boldsymbol{\varphi}}_i^\top \dot{\boldsymbol{\Delta}}_i + \dot{f}_{yi}.$$

Since $\boldsymbol{\Delta}_i$ is generated by a stable filter $G_{ii}(s)$, its derivative $\dot{\boldsymbol{\Delta}}_i(t)$ is also bounded. In the absence of $\mathbf{f}$, $f_{yi}(t)$ is exponentially decaying, hence $\dot{f}_{yi}(t)$ is bounded. Furthermore, from the proof of Property T.1.1, it is established that $\tilde{\boldsymbol{\varphi}}_i$ and $\dot{\tilde{\boldsymbol{\varphi}}}_i$ are bounded. Since $\dot{\hat{\boldsymbol{\varphi}}}_i = -\dot{\tilde{\boldsymbol{\varphi}}}_i$, the boundedness of $\dot{\hat{\boldsymbol{\varphi}}}_i$ directly follows. Consequently, $\dot{\bar{y}}_i$ is bounded, which implies that $\bar{y}_i$ is uniformly continuous.

Since $\bar{y}_i$ is uniformly continuous and $\int_0^\infty \bar{y}_i^2(\tau)d\tau < \infty$, by Barbalat's lemma [1] it follows that $\bar{y}_i \to 0$ as $t \to \infty$. Considering expressions (16), (19) and (20), and taking into account that $\mathbf{f} \equiv 0$, we can conclude that $\mathbf{y}$ also tends to zero as $t \to \infty$. Therefore, Property T.1.2 is proved. ■

B. Adaptation algorithm with memory regressor extension

In this study, an adaptive algorithm with Memory Regressor Extension (MRE) is employed to improve the convergence rate of signals in the closed-loop system [1].

Proposition 2. Assume that Assumptions 1 and 2 are satisfied. If the matrix $\mathbf{B}^*$ is nonsingular and $\mathbf{f} \equiv 0$ then the adaptive algorithm with MRE can be constructed independently for each control channel for system (6) as follows:

$$\dot{\hat{\boldsymbol{\varphi}}}_i = \gamma_i(\mathbf{Y}_i - \boldsymbol{\Omega}_i\hat{\boldsymbol{\varphi}}_i),\ i = 1, 2, \ldots, m. \quad (31)$$

Here, $\mathbf{Y}_i = H(s)[\boldsymbol{\Delta}_i\hat{\varepsilon}_i]$, $\boldsymbol{\Omega}_i = H(s)[\boldsymbol{\Delta}_i\boldsymbol{\Delta}_i^\top]$, $\hat{\varepsilon}_i = y_i - G_{ii}[u_{di}]$, $\boldsymbol{\Delta}_i = G_{ii}(s)[\hat{\boldsymbol{\xi}}(t)]$, where y_i is a component of y, $\gamma_i > 0$ is the adaptation gain, and $H(s) = \dfrac{1}{\beta s + 1}$, $\beta > 0$.

Proof. Since $\mathbf{f}(t) \equiv 0$, expression (16) can be rewritten as follows:

$$\mathbf{y}(t) = \mathbf{G}_u(s)[\tilde{\boldsymbol{\eta}}^\top\hat{\boldsymbol{\xi}}(t)]. \quad (32)$$

An extended variable is constructed in the form

$$\hat{\boldsymbol{\varepsilon}} = \mathbf{y}(t) + \boldsymbol{\Delta}^\top\hat{\boldsymbol{\varphi}}. \quad (33)$$

By taking into account (14), (32), equation (33) is rewritten:

$$\hat{\boldsymbol{\varepsilon}} = \boldsymbol{\Delta}^\top\boldsymbol{\varphi}, \text{ or } \hat{\boldsymbol{\varepsilon}} = \mathbf{y} - \mathbf{G}_u(s)[\mathbf{u}_d(t)]. \quad (34)$$

We multiply both sides of equation (34) by $\boldsymbol{\Delta} = \mathbf{G}_u(s)[\hat{\boldsymbol{\xi}}(t)]$ and applying the transfer function $H(s)$, we obtain the following expression:

$$\mathbf{Y} = \boldsymbol{\Omega}\boldsymbol{\varphi}, \quad (35)$$

where $H(s) = \dfrac{1}{\beta s + 1}$ with $\beta > 0$, $\mathbf{Y} = H(s)[\boldsymbol{\Delta}\hat{\varepsilon}]$ and $\boldsymbol{\Omega} = H(s)[\boldsymbol{\Delta}\boldsymbol{\Delta}^\top]$.

Based on equation (35), the adaptive algorithm is constructed as follows:

$$\dot{\hat{\boldsymbol{\varphi}}} = \gamma(\mathbf{Y} - \boldsymbol{\Omega}\hat{\boldsymbol{\varphi}}), \quad (36)$$

where $\gamma > 0$ is the adaptation gain.

Since the matrix $\boldsymbol{\Delta}$ has a block-diagonal structure, the adaptive algorithm can therefore be constructed independently for each channel in the form (31). ■

Therefore, the following theorem is formulated and proved by analogy with the case of SISO systems presented in [1].

Theorem 2. The robust algorithm with MRE is constructed as follows:

$$\dot{\hat{\boldsymbol{\varphi}}}_i = \gamma_i(\mathbf{Y}_i - \boldsymbol{\Omega}_i\hat{\boldsymbol{\varphi}}_i) - \gamma_i\sigma(\hat{\boldsymbol{\varphi}}_i)\hat{\boldsymbol{\varphi}}_i,\ i = 1, 2, \ldots, m, \quad (37)$$

where $\sigma(\hat{\boldsymbol{\varphi}}_i)$ defined by (23), ensures that:

T.2.1. All signals in the closed-loop system are bounded, and if $\varphi_i^* \geq ||\boldsymbol{\varphi}_i||$, then the output vector norm $||\mathbf{y}||$ is of the order of $||\bar{\mathbf{f}}|| > ||\mathbf{f}(t)||$ in the mean-square sense, i.e., $||\mathbf{y}|| \in \mathcal{MS}(||\bar{\mathbf{f}}||)$.

T.2.2. If $\varphi_i^* \geq ||\boldsymbol{\varphi}_i||$, and $\mathbf{f} \equiv 0$, then the output signal asymptotically converges to zero, i.e., $\mathbf{y}(t) \to 0$ as $t \to \infty$.

Proof. From expression (37), the error model can be represented as

$$\dot{\tilde{\boldsymbol{\varphi}}}_i = -\gamma_i\boldsymbol{\Omega}_i\tilde{\boldsymbol{\varphi}}_i - \gamma_i H(s)[\boldsymbol{\Delta}_i f_{yi}] + \sigma(\hat{\boldsymbol{\varphi}}_i)\hat{\boldsymbol{\varphi}}_i.$$

Choose the Lyapunov function as $V = \dfrac{1}{2\gamma_i}\tilde{\boldsymbol{\varphi}}_i^\top\tilde{\boldsymbol{\varphi}}_i$. Taking the time derivative yields:

$$\dot{V} = -\tilde{\boldsymbol{\varphi}}_i^\top\boldsymbol{\Omega}_i\tilde{\boldsymbol{\varphi}}_i - \tilde{\boldsymbol{\varphi}}_i^\top H(s)[\boldsymbol{\Delta}_i f_{yi}] + \sigma\tilde{\boldsymbol{\varphi}}_i^\top\hat{\boldsymbol{\varphi}}_i.$$

Using the Schwartz inequality, we can write [1]:

$$\dot{V} \le -\varepsilon_\Omega^2 + \bar{f}_{yi}|\varepsilon_\Omega| + \sigma\tilde{\boldsymbol{\varphi}}_i^{\top}\hat{\boldsymbol{\varphi}}_i,$$

where $\varepsilon_\Omega = (\tilde{\boldsymbol{\varphi}}_i^{\top}\boldsymbol{\Omega}_i\tilde{\boldsymbol{\varphi}}_i)^{1/2}$.

The latter expression is in the form of (25), which allows extending the results of Theorem 1 to the adaptation algorithm with MRE [1]. ■

Numerical example

We consider a third-order unstable linear system (1) with

$$\mathbf{A} = \begin{bmatrix} -3.2 & -1.7 & -3 \\ 2.8 & 1.3 & 4 \\ 0.3 & 0.3 & -0.5 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -2 & 1 \\ 2 & -1 \\ 1 & 0 \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 2 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 3 & -3 \\ 1 & -2 \\ 2 & -2 \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}.$$

Assume that the unknown external disturbances consist of

$$\mathbf{v}(t) = \begin{cases} 3\sin(2t) \\ 3\sin(3t) + 2, \end{cases} \text{ and}$$

$$f_1(t) = f_2(t) = \begin{cases} 0.1\sin(t)\sin(10t), \text{ if } 0 \le t \le \bar{t} \text{ sec} \\ 0, \text{ if } t > \bar{t} \text{ sec}. \end{cases}$$

Because $v_1 = 3\sin(2t)$ and $v_2 = 3\sin(3t) + 2$, the disturbance observer (10) and (11) is constructed with the following matrices:

$$\mathbf{G}_{v1} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}, \quad \mathbf{I}_{v1} = \begin{bmatrix} 0 \\ 2 \end{bmatrix},$$

$$\mathbf{G}_{v2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}, \quad \mathbf{I}_{v2} = \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix}.$$

Based on equation (5), we determine $\rho_1 = 2$ and $\rho_2 = 1$ for the respective outputs. Choosing the poles for the first channel as $\lambda_{11} = -6$, $\lambda_{12} = -2$ and for the second channel as $\lambda_{21} = -3$, the decoupling control law (4) is designed using the following matrices:

$$\mathbf{F} = \begin{bmatrix} 6 & 0 \\ 0 & 3 \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} 9.26 & 9.26 & 7.1 \\ 3 & 1.5 & 3 \end{bmatrix}.$$

For the adaptation algorithm with MRE (36), we choose $H(s) = \dfrac{1}{0.2s + 1}$.

Fig. 1 and 2 present the simulation results obtained using the adaptation algorithm with MRE for the case $\mathbf{v}(t) \equiv 0$. The case $\sigma(\hat{\boldsymbol{\varphi}}_1) \equiv 0$ and $\sigma(\hat{\boldsymbol{\varphi}}_2) \equiv 0$, corresponding to the application of algorithm (31) without the σ-modification term, is illustrated in Fig. 1, whereas Fig. 2 presents the case in which algorithm (37) is applied with the σ-modification term, using $\varphi_1^* = 2$ and $\varphi_2^* = 0.5$. Fig. 1 illustrates the unbounded parametric drift of the system. This phenomenon can be avoided by applying algorithm (37), as shown in the right plot of Fig. 2.

Simulation results for the closed-loop systems employing the robust modification of the gradient-based adaptation algorithm and the adaptation algorithm with MRE are presented in Fig. 3 and 4, with $\varphi_1^* = 25$ and $\varphi_2^* = 15$. The simulation results shown in Fig. 3 and 4 were obtained for $\bar{t} = 80$ s and $\bar{t} = 30$ s, respectively. It can be seen from the simulation results that, in the presence of nonzero irregular component $\mathbf{f}(t)$, the plant outputs approach small steady-state values.

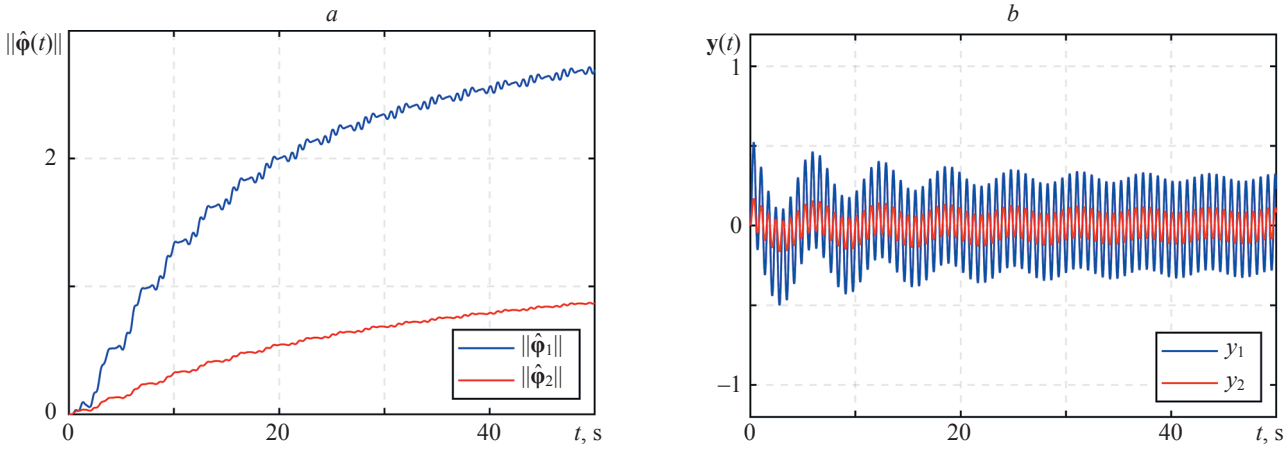

Fig. 1. Response of the adaptive system to the disturbance $\mathbf{v}_f(t) = \mathbf{f}(t)$ using algorithm (31) without the σ-modification: transient process of the adjusted parameter $\hat{\boldsymbol{\varphi}}$ (*a*) and of the system output $\mathbf{y}(t)$ (*b*)

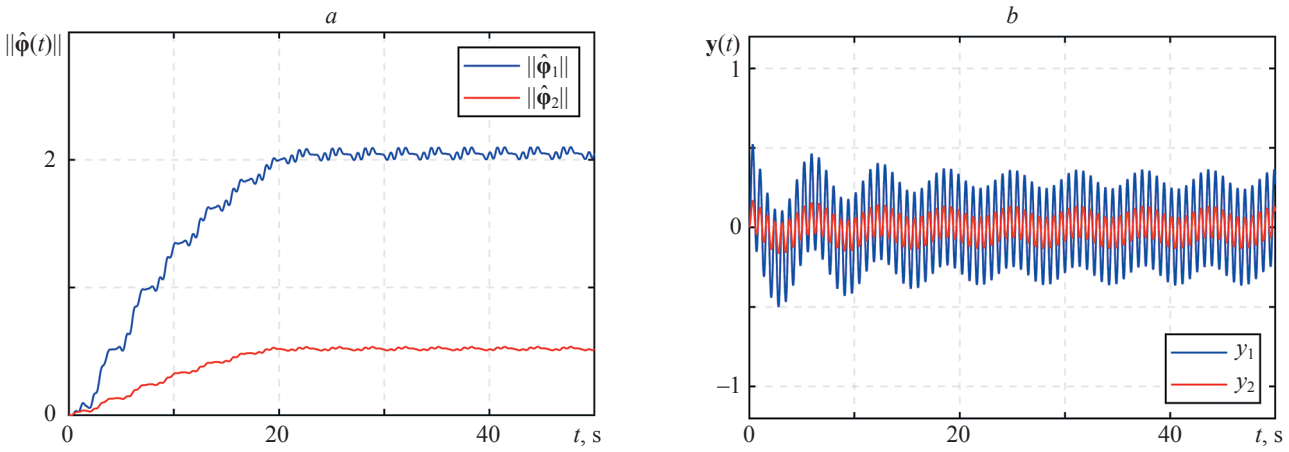

Fig. 2. Response of the robust adaptive disturbance compensation system to the disturbance $\mathbf{v}_f(t) = \mathbf{f}(t)$ using algorithm (37) with the σ-modification, where $\varphi_1^* = 2$ and $\varphi_2^* = 0.5$: transient process of the adjusted parameter $\hat{\boldsymbol{\varphi}}$ (*a*) and of the system output $\mathbf{y}(t)$ (*b*)

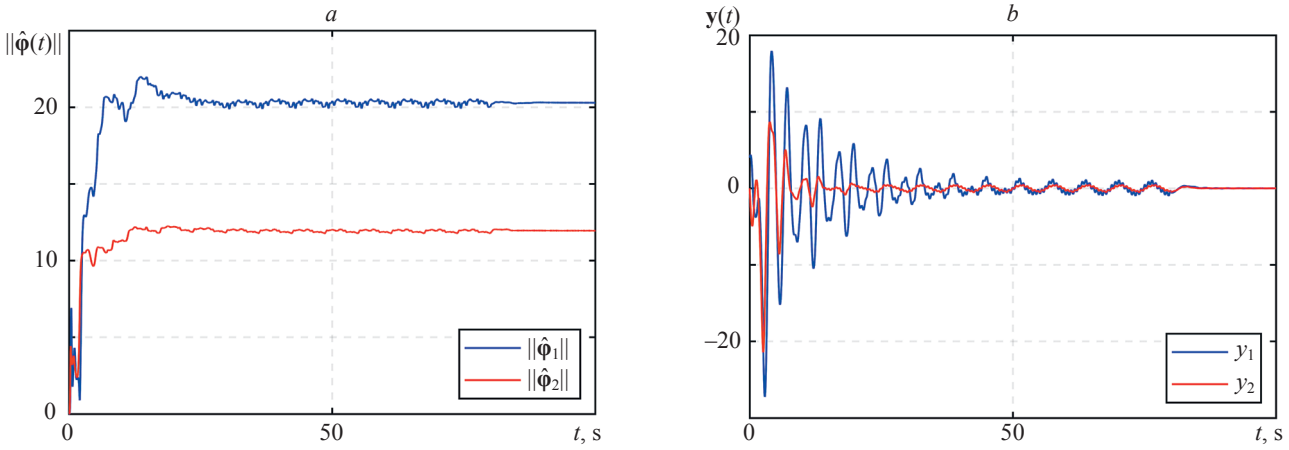

Fig. 3. Adaptive disturbance compensation for the disturbance $\mathbf{v}_f(t)$ containing an irregular component using the gradient-based adaptation algorithm: transient process of the adjusted parameter $\hat{\boldsymbol{\varphi}}$ (*a*) and of the system output $\mathbf{y}(t)$ (*b*)

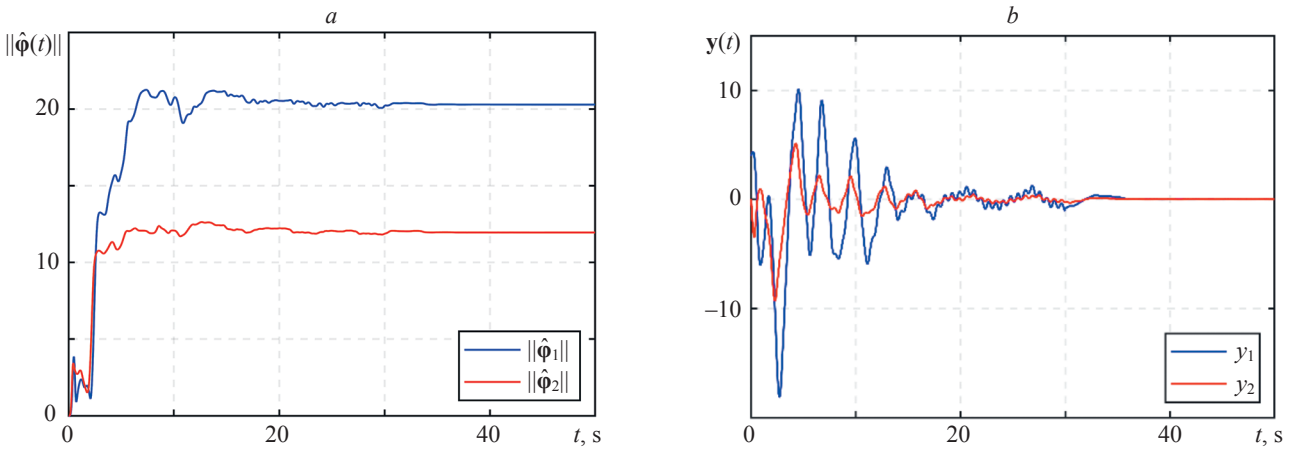

Fig. 4. Adaptive disturbance compensation for the disturbance $\mathbf{v}_f(t)$ containing an irregular component using the robust algorithm with Memory Regressor Extension: transient process of the adjusted parameter $\hat{\boldsymbol{\varphi}}$ (*a*) and of the system output $\mathbf{y}(t)$ (*b*)

Conclusion

This paper addresses the problem of robust adaptive disturbance compensation for a class of Multi-Input Multi-Output Linear Time-Invariant Systems in the presence of unknown external disturbances, which may be unmatched with the control inputs and may affect both the system states and outputs. The proposed approach combines the state-feedback decoupling method with the internal model-based disturbance compensation technique. As a result, independent adaptation algorithms are constructed for each control channel, allowing the adaptive laws to be designed robustly for every channel separately. Consequently, the proposed algorithm guarantees the boundedness of all closed-loop system signals, while the norm of the output vector remains of the same order as the disturbance in the mean-square sense.

Furthermore, with the proposed approach, the adaptive algorithm is designed independently for each channel, thereby reducing the number of tuning parameters, decreasing the computational complexity, and making the adaptive algorithm easier to tune.

References

1. Nikiforov V.O., Gerasimov D.N. *Adaptive Regulation: Reference Tracking and Disturbance Rejection*. Springer, 2022, 374 p. doi: 10.1007/978-3-030-96091-9
2. Pyrkin A., Isidori A. Adaptive output regulation of right-invertible MIMO LTI systems, with application to vessel motion control. *European Journal of Control*, 2019, vol. 46, pp. 63–79. doi: 10.1016/j.ejcon.2018.04.001
3. Nikiforov V.O., Paramonov A.V., Gerasimov D.N. Adaptive control algorithms in MIMO linear systems with control delay. *Automation and Remote Control*, 2020, vol. 81, no. 6, pp. 1091–1106. doi: 10.1134/S0005117920060107
4. Obregon-Pulido G., Castillo-Toledo B., Loukianov A.G. A structurally stable globally adaptive internal model regulator for MIMO linear systems. *IEEE Transactions on Automatic Control*, 2011, vol. 56, no. 1, pp. 160–165. doi: 10.1109/TAC.2010.2090409
5. Borisov O., Isidori A., Pyrkin A. Adaptive output regulation of MIMO LTI systems with unmodeled input dynamics. *Proc. of the 62nd IEEE Conference on Decision and Control (CDC)*, 2023, pp. 1537–1542. doi: 10.1109/CDC49753.2023.10383343
6. Marino R., Tomei P. Output regulation for linear systems via adaptive internal model. *IEEE Transactions on Automatic Control*, 2003, vol. 48, no. 12, pp. 2199–2202. doi: 10.1109/TAC.2003.820143
7. Bobtsov A.A. Output control algorithm with the compensation of biased harmonic disturbances. *Automation and Remote Control*, 2008, vol. 69, no. 8, pp. 1289–1296. doi: 10.1134/S000511790808002X
8. Bobtsov A.A., Pyrkin A.A. Compensation of unknown sinusoidal disturbances in linear plants of arbitrary relative degree. *Automation and Remote Control*, 2009, vol. 70, no. 3, pp. 449–456. doi: 10.1134/S0005117909030102
9. Yilmaz C.T., Basturk H.I. Output feedback control for unknown LTI systems driven by unknown periodic disturbances. *Automatica*, 2019, vol. 99, pp. 112–119. doi: 10.1016/j.automatica.2018.10.020
10. Marino R., Santosuosso G.L. Regulation of linear systems with unknown exosystems of uncertain order. *IEEE Transactions on Automatic Control*, 2007, vol. 52, no. 2, pp. 352–359. doi: 10.1109/TAC.2006.890376
11. Ortega R., Tang Y. Robustness of adaptive controllers-A survey. *Automatica*, 1989, vol. 25, no. 5, pp. 651–677. doi: 10.1016/0005-1098(89)90023-X
12. Ioannou P.A., Kokotovic P.V. Instability analysis and improvement of robustness of adaptive control. *Automatica*, 1984, vol. 20, no. 5, pp. 583–594. doi: 10.1016/0005-1098(84)90009-8
13. Narendra K.S., Annaswamy A.M. *Stable Adaptive Systems*. Prentice Hall, 1989, 494 p.
14. Nikiforov V.O. Robust high-order tuner of simplified structure. *Automatica*, 1999, vol. 35, no. 8, pp. 1409–1415. doi: 10.1016/S0005-1098(99)00051-5
15. Falb P., Wolovich W. Decoupling in the design and synthesis of multivariable control systems. *IEEE Transactions on Automatic Control*, 1967, vol. 12, no. 6, pp. 651–659. doi: 10.1109/TAC.1967.1098737
16. Wang Q.-G. *Decoupling Control*. Springer, 2003, 384 p. doi: 10.1007/3-540-46151-5
17. Nikiforov V.O. Adaptive servocompensation of input disturbances. *IFAC Proceedings Volumes*, 1996, vol. 29, no. 1, pp. 5114–5119. doi: 10.1016/S1474-6670(17)58492-X

Литература

1. Nikiforov V.O., Gerasimov D.N. Adaptive Regulation: Reference Tracking and Disturbance Rejection. Springer, 2022. 374 p. doi: 10.1007/978-3-030-96091-9
2. Pyrkin A., Isidori A. Adaptive output regulation of right-invertible MIMO LTI systems, with application to vessel motion control // European Journal of Control. 2019. V. 46. P. 63–79. doi: 10.1016/j.ejcon.2018.04.001
3. Nikiforov V.O., Paramonov A.V., Gerasimov D.N. Adaptive control algorithms in MIMO linear systems with control delay // Automation and Remote Control. 2020. V. 81. N 6. P. 1091–1106. doi: 10.1134/S0005117920060107
4. Obregon-Pulido G., Castillo-Toledo B., Loukianov A.G. A structurally stable globally adaptive internal model regulator for MIMO linear systems // IEEE Transactions on Automatic Control. 2011. V. 56. N 1. P. 160–165. doi: 10.1109/TAC.2010.2090409
5. Borisov O., Isidori A., Pyrkin A. Adaptive output regulation of MIMO LTI systems with unmodeled input dynamics // Proc. of the 62nd IEEE Conference on Decision and Control (CDC). 2023. P. 1537–1542. doi: 10.1109/CDC49753.2023.10383343
6. Marino R., Tomei P. Output regulation for linear systems via adaptive internal model // IEEE Transactions on Automatic Control. 2003. V. 48. N 12. P. 2199–2202. doi: 10.1109/TAC.2003.820143
7. Bobtsov A.A. Output control algorithm with the compensation of biased harmonic disturbances // Automation and Remote Control. 2008. V. 69. N 8. P. 1289–1296. doi: 10.1134/S000511790808002X
8. Bobtsov A.A., Pyrkin A.A. Compensation of unknown sinusoidal disturbances in linear plants of arbitrary relative degree // Automation and Remote Control. 2009. V. 70. N 3. P. 449–456. doi: 10.1134/S0005117909030102
9. Yilmaz C.T., Basturk H.I. Output feedback control for unknown LTI systems driven by unknown periodic disturbances // Automatica. 2019. V. 99. P. 112–119. doi: 10.1016/j.automatica.2018.10.020
10. Marino R., Santosuosso G.L. Regulation of linear systems with unknown exosystems of uncertain order // IEEE Transactions on Automatic Control. 2007. V. 52. N 2. P. 352–359. doi: 10.1109/TAC.2006.890376
11. Ortega R., Tang Y. Robustness of adaptive controllers-A survey // Automatica. 1989. V. 25. N 5. P. 651–677. doi: 10.1016/0005-1098(89)90023-X
12. Ioannou P.A., Kokotovic P.V. Instability analysis and improvement of robustness of adaptive control // Automatica. 1984. V. 20. N 5. P. 583–594. doi: 10.1016/0005-1098(84)90009-8
13. Narendra K.S., Annaswamy A.M. Stable Adaptive Systems. Prentice Hall, 1989. 494 p.
14. Nikiforov V.O. Robust high-order tuner of simplified structure // Automatica. 1999. V. 35. N 8. P. 1409–1415. doi: 10.1016/S0005-1098(99)00051-5
15. Falb P., Wolovich W. Decoupling in the design and synthesis of multivariable control systems // IEEE Transactions on Automatic Control. 1967. V. 12. N 6. P. 651–659. doi: 10.1109/TAC.1967.1098737
16. Wang Q.-G. Decoupling Control. Springer, 2003. 384 p. doi: 10.1007/3-540-46151-5
17. Nikiforov V.O. Adaptive servocompensation of input disturbances // IFAC Proceedings Volumes. 1996. V. 29. N 1. P. 5114–5119. doi: 10.1016/S1474-6670(17)58492-X

18. Nikiforov V.O. Adaptive non-linear tracking with complete compensation of unknown disturbances. *European Journal of Control*, 1998, vol. 4, no. 2, pp. 132–139. doi: 10.1016/S0947-3580(98)70107-4
19. Nikiforov V.O., Paramonov A.V., Gerasimov D.N. Adaptive compensation of unmatched disturbances in unstable MIMO LTI plants with distinct input delays. *IFAC-PapersOnLine*, 2023, vol. 56, no. 2, pp. 9179–9184. doi: 10.1016/j.ifacol.2023.10.159
20. Marino R., Tomei P. Disturbance cancellation for linear systems by adaptive internal models. *Automatica*, 2013, vol. 49, no. 5, pp. 1494–1500. doi: 10.1016/j.automatica.2013.02.011
21. Isidori A. *Lectures in Feedback Design for Multivariable Systems*. Springer, 2017, 424 p. doi: 10.1007/978-3-319-42031-8
22. Ioannou P.A., Sun J. *Robust Adaptive Control*. PTR Prentice-Hall, 1996, 825 p.

18. Nikiforov V.O. Adaptive non-linear tracking with complete compensation of unknown disturbances // European Journal of Control. 1998. V. 4. N 2. P. 132–139. doi: 10.1016/S0947-3580(98)70107-4
19. Nikiforov V.O., Paramonov A.V., Gerasimov D.N. Adaptive compensation of unmatched disturbances in unstable MIMO LTI plants with distinct input delays // IFAC-PapersOnLine. 2023. V. 56. N 2. P. 9179–9184. doi: 10.1016/j.ifacol.2023.10.159
20. Marino R., Tomei P. Disturbance cancellation for linear systems by adaptive internal models // Automatica. 2013. V. 49. N 5. P. 1494–1500. doi: 10.1016/j.automatica.2013.02.011
21. Isidori A. Lectures in Feedback Design for Multivariable Systems. Springer, 2017. 424 p. doi: 10.1007/978-3-319-42031-8
22. Ioannou P.A., Sun J. Robust Adaptive Control. PTR Prentice-Hall, 1996. 825 p.

Authors

Cong Vinh Tu — PhD Student, ITMO University, Saint Petersburg, 197101, Russian Federation, https://orcid.org/0009-0005-5106-8560, congvinhvkdn@gmail.com
Natalia A. Dudarenko — PhD, Associate Professor, Associate Professor, ITMO University, Saint Petersburg, 197101, Russian Federation, SC 36662185600, https://orcid.org/0000-0002-3553-0584, dudarenko@itmo.ru

Авторы

Ты Конг Винь — аспирант, Университет ИТМО, Санкт-Петербург, 197101, Российская Федерация, https://orcid.org/0009-0005-5106-8560, congvinhvkdn@gmail.com
Дударенко Наталия Александровна — кандидат технических наук, доцент, доцент, Университет ИТМО, Санкт-Петербург, 197101, Российская Федерация, SC 36662185600, https://orcid.org/0000-0002-3553-0584, dudarenko@itmo.ru

Received 19.06.2026
Approved after reviewing 24.07.2026
Accepted 20.09.2026

Статья поступила в редакцию 19.06.2026
Одобрена после рецензирования 24.07.2026
Принята к печати 20.09.2026

Работа доступна по лицензии
Creative Commons
«Attribution-NonCommercial»